NASA Contractor Report 2855

Equation Modifying Program, L219 (EQMOD)

Volume I: Engineering and Usage

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1.0 SUMMARY

This document describes the analysis and use of L219 (EQMOD), a digital computer program to modify matrices according to specific instructions. The particular field of application of the program is to modify the theoretical equations of motion and load equations generated by the DYLOFLEX programs Equations of Motion, L217 (EOM), and Load Equations, L218 (LOADS), respectively.

The equations of motion and load equation coefficient matrices must be formulated outside of L219 (EQMOD) and read as program input data from magnetic files. These matrices can then be modified according to specific card input instructions, which allow the user to:

- Use scalar multipliers.
- Replace or increment individual matrix elements.
- Add sensor equations to the equations of motion.
- Add the definition of the active control system to the equations of motion.
- Replace the theoretical rigid body and control stability derivatives in the equations of motion with those calculated by FLEXSTAB or other external means.
- Transform the equations of motion and load equations from the inertia axis system to the body-fixed axis system.
- Prepare matrix coefficients in a form useable in the Random Harmonic Analysis Program, L221.
- Prepare matrix coefficients in a form useable in the Linear System Analysis Program, QR, to include:
    - Equations of motion with and without Wagner lift growth functions
    - Equations of motion and load equations combined for a time history solution

The modified equations of motion and load equation matrix coefficients are saved on magnetic files for interfacing with solution programs L221 (TEV156) and QR.

Limitations imposed on the matrices are:

- The equations must be generated using the inertia axis system and a straight and level reference flight condition.
- The vertical and lateral gusts are uncoupled (i.e., there are at most three rigid body degrees of freedom represented in the matrix coefficients for either the vertical or lateral gust analysis).

2.0 INTRODUCTION

The computer program L219 (EQMOD) may be used as either a standalone program or as a module of a program system called DYLOFLEX, which was developed for NASA under contract NAS1-13918 (ref. 1). Because of the DYLOFLEX contract requirements developed in reference 2, a program was needed to modify the equations of motion matrices generated by L217 (EOM) (ref. 3) and the load equation matrices generated by L218 (LOADS) (ref. 4).

Modifications to the equations are needed when externally developed stability derivatives, such as experimental, or those developed in FLEXSTAB (ref. 5), are to be incorporated in the theoretical equations of motion. Other modifications such as varying the dynamic pressure and velocity, may be desired for parameter studies. These can be performed by matrix scalar modifications rather than regenerating the equations of motion and load equations with new aerodynamics.

3.0 SYMBOLS AND ABBREVIATIONS

With the exception of section 6.3 (card input), all the items that appear in this document are listed below.

a, b, c, d, e, f	Dummy coefficients used for the mathematical description of a typical control system
a_1, b_1	Wagner indicial lift growth function coefficients
A_{ij}	Time variant coefficients (functions of ϕ_1, θ_1, c_1) used in the development of the equations of motion.
b	Reference span, length
$\{C_3\}$	Matrix of generalized forcing function coefficients
$\{\overline{C}_3\}$	Matrix of load coefficients due to the excitation forces
C_D	Aerodynamic drag coefficient
C_L	Aerodynamic lift coefficient
C_ℓ	Aerodynamic rolling moment coefficient
C_m	Aerodynamic pitching moment coefficient
C_n	Aerodynamic yawing moment coefficient
C_y	Aerodynamic side force coefficient
$\overline{c}$	Reference chord, length
D	Aerodynamic drag force
$\overline{D}$	Generalized structural damping associated with elastic coordinates
F_x, F_y, F_z	External forces defined in the body-fixed axis system.
$F_{x'}, F_{y'}, F_{z'}$	External forces defined in the interia axis system
$\{f_\ell\}$	Matrix of streamwise distances from points first encountering the gust to the points encountering the gust later
g	Gravity constant
h	Acceleration at a particular sensor location

I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, I_{yz}	Airplane inertias
$\overline{K}$	Generalized structural stiffness associated with the elastic coordinates
L	Aerodynamic lift force
ℓ	Aerodynamic rolling moment
$[M_1]$, $[M_2]$, $[M_3]$	Generalized structural stiffness, damping, and inertia matrix coefficients
$[\overline{M}_1]$, $[\overline{M}_2]$, $[\overline{M}_3]$	Load coefficient matrices (nonaerodynamic) of the generalized coordinate displacement rate, and acceleration, respectively
$[M_4]$, $[M_5]$	Generalized aerodynamic stiffness and damping matrix coefficients
$[\overline{M}_4]$, $[\overline{M}_5]$	Load coefficient matrices (aerodynamic) of the generalized coordinate displacements and rates
M	Airplane mass
$\overline{M}$	Generalized mass associated with the elastic coordinates
M_x, M_y, M_z	External moments defined in the body-fixed axis system
$M_{x'}$, $M_{y'}$, $M_{z'}$	External moments defined in the inertia axis system
m	Aerodynamic pitching moment
n	Aerodynamic yawing moment
p, q, r	Rigid body roll, pitch, and yaw rates defined in the body-fixed axis system
q	Generalized coordinate
$\overline{q}$	Dynamic pressure
S_W	Wing reference area
s	Laplace variable
$U_1 = V_T$	True velocity

u, v, w	Airplane velocities defined in the body-fixed axis system
u_{e_a}, u_{e_s}	Perturbation values of antisymmetric and symmetric elastic coordinates
W_1	Airplane vertical velocity in 1-g flight
x, y, z	Linear displacement in the body-fixed axis system coordinates
x′, y′, z′	Linear displacements in the inertia axis system
x_{cg}, z_{cg}	Coordinates of the airplane's center of gravity in the body-fixed axis system
Y	Aerodynamic side force
α_g	Gust angle of attack
α_1	Trim angle of attack
α_1,β_1	Exponential coefficients for the Wagner indicial lift growth function
$\Phi^{(t)}$	Wagner indicial lift growth function
$[\tilde{\phi}]$	Forcing function matrix when accounting for the gradual penetration of the gust
$[\tilde{\tilde{\phi}}]$	Matrix of load coefficients due to the excitation forces when accounting for the gradual penetration of the gust
$\lfloor \phi h \rfloor$	Modal displacements at the sensor location
ϕ,θ,ψ	Airplane angular displacements defined in the body-fixed axis system
ϕ',θ',ψ'	Airplane angular displacements defined in the inertia axis system
δ_c	The amount of rotation experienced by the control surface
δ_c'	The amount of control surface rotating commanded by the stability augmentation system
$\psi(t)$	Küssner indicial lift growth function

4.0 ENGINEERING AND MATHEMATICAL DESCRIPTION

The equations of motion developed in L217 (ref. 3) are represented in the form:

$$[M_1]\{q\} + [M_2]\{\dot{q}\} + [M_3]\{\ddot{q}\} + [M_4]\{\dot{q}\} * \Phi + [M_5]\{\ddot{q}\} * \Phi = \{C_3\}\,\dot{\alpha}_g * \psi \quad (1)$$

where:

M, C = Appropriate matrix coefficients[1]

$q, \dot{q}, \ddot{q}$ = Generalized coordinates and their time deratives, including SAS degrees of freedom

α_g = Gust angle

Φ = Wagner function (equal one for no lift growth)

ψ = Kussner function (equal one for no lift growth)

* Indicates convolution

With gust penetration, the excitation function, $\{C_3\}\,\dot{\alpha}_g$, of equation (1) is frequency dependent and is defined as:

$$\{C_3\}\,\dot{\alpha}_g = [\tilde{\phi}]\cos(\Omega\{f_\ell\}) - i\,[\tilde{\phi}]\sin(\Omega\{f_\ell\}) \quad (2)$$

where:

Ω = ω/V_T, spatial frequency (rad/unit length)

$\{f_\ell\}$ = Matrix of streamwise distances from points first encountering gust to the points encountering the gust later

$[\tilde{\phi}]$ = Contribution of the lifting panels due to the gust force at designated gradual penetration load stations

Relating these matrices in a physical sense, $[M_1]$, $[M_2]$, and $[M_3]$ are usually associated with the generalized structural forces and the active control system definition. $[M_4]$ and $[M_5]$ are usually associated with the generalized aerodynamic forces and $\{C_3\}$ with the generalized excitation (gust) force.

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[1]These matrix coefficients can be either constant or nonconstant (frequency dependent) coefficients.

The load equations developed in L218 (ref. 4) follow the same format as the equations of motion.

$$\{LOAD\} = [\bar{M}_1]\ \{q\} + [\bar{M}_2]\ \{\dot{q}\} + [\bar{M}_3]\ \{\ddot{q}\} + [\bar{M}_4]\ \{\dot{q}\} * \Phi + [\bar{M}_5]\ \{\ddot{q}\} * \Phi + (\{\bar{C}_3\}\ \dot{\alpha}_g * \psi) \quad (3)$$

where:

$[\overline{M}_1]$, $[\overline{M}_2]$, $[\overline{M}_3]$ = Load matrix coefficients of the generalized coordinate displacement, rate, and acceleration, respectively

$[\overline{M}_4]$, $[\overline{M}_5]$ = Load matrix coefficients of the generalized coordinate rate and acceleration convoluted with the Wagner function

$\{\overline{C}_3\}$ = Load matrix coefficient of the excitation function convoluted with the Küssner function

With gust penetration, the excitation function, $\{\overline{C}_3\}\,\alpha_g$, of equation (3) is frequency dependent and is defined in a manner similar to equation (2):

$$\{\bar{C}_3\}\ \dot{\alpha}_g = [\bar{\bar{\phi}}]\ \cos(\Omega\ \{f_\ell\}) - i\ [\bar{\bar{\phi}}]\ \sin(\Omega\ \{f_\ell\}) \quad (4)$$

where:

$[\bar{\bar{\phi}}]$ = contribution of the lifting panels due to gust forces at designated gradual penetration load stations to aircraft loads.

Relating these matrices in a physical sense, $[\overline{M}_1]$, $[\overline{M}_2]$, and $[\overline{M}_3]$ are usually associated with the load resulting from structural response; $[\overline{M}_4]$ and $[\overline{M}_5]$ are usually associated with the load resulting from aerodynamic response forces; and $\{\overline{C}_3\}$ with load resulting from the gust excitation force.

EQMOD offers an analyst the capability to alter the matrix coefficients of equations (1) to (4). Section 4.1 discusses the option to replace the appropriate theoretical aerodynamic terms in the M_4, M_5 and C_3 matrices with airplane stability derivatives obtained from external sources such as flight or wind tunnel data. Section 4.2 details the option of incorporating the equations describing an airplane's active control system into the basic equations of motion and load equations (all M, $\overline{M}$, C and $\overline{C}$ matrices). The option to scalar multiply matrices or replace or increment individual matrix elements is presented in section 4.3. The option to transform the equations of motion of equation (1) into a form from which a stability analysis can be performed by solving for the eigenvalues of the equation is explained in section 4.4. Finally, section 4.5 discusses the changes of the matrix coefficients of equations (1) and (3) to transform from the inertial axes to the body-fixed axes.

4.1 STABILITY DERIVATIVES

Developing the equations of motion for straight and level flight in the inertia axis system will result in some rigid body generalized coordinates acting at a vehicle reference point, representing rigid-body forward displacement (x), vertical displacement (z), and pitch (θ) for symmetric flight conditions and lateral displacement (y), roll (ϕ), and yaw (ψ) for antisymmetric flight conditions. Some of the generalized coordinates may also represent rigid rotations about various control surface hinge lines. Embedded in the generalized aerodynamic and gust matrix coefficients of $[M_4]$, $[M_5]$, and $\{C_3\}$ are the total airplane theoretical aerodynamic forces and moments due to the airplane's rigid body motions, control surface deflections, and gust angle of attack. For a symmetric analysis, these total airplane forces and moments correspond to the airplane's lift, drag, and pitching moment defined in the inertial axis system. Similarly, for an antisymmetric analysis, these forces and moments correspond to the airplane's side force, rolling, and yawing moments.

These total airplane forces and moments in the inertial axis system can be related to appropriate airplane stability derivatives, usually defined in a particular stability axis system. This relationship is defined in detail in appendix A. A summary of the relationship between the stability derivatives and the appropriate total airplane force and moment elements of $[M_4]$, $[M_5]$, and $[C_3]$ are shown in figures 1, 2 and 3. If these derivatives are available from wind tunnel results, flight test results, or any other source, of if they are calculated in FLEXSTAB (ref. 5), they can be used to calculate the appropriate matrix coefficients and used in preference to the theoretical coefficients calculated in L217 (EOM).

Since elastic modal degrees of freedom (elastic generalized coordinates) are included in the dynamic analysis, the stability derivatives used in the equations of motion should be only rigid body stability derivatives. The aeroelastic effects that are represented by elastic increments to the rigid body stability derivatives are reflected in the equations of motion through the elastic modal representation. However, since DYLOFLEX in general does not consider panel aerodynamics that are not perpendicular to lifting surfaces, no aeroelastic effects are represented in the forward generalized coordinate displacement. Consequently, the symmetric representations in figures 1 and 3 use both the rigid stability derivatives and the elastic increment to the rigid stability derivative in calculating the generalized matrix coefficients for the forward (x) generalized coordinate to obtain aeroelastic effects for that degree of freedom in the dynamic analysis.

In addition, the development of the expressions in figures 1 through 3 assumes that the stability derivatives and the rigid body motions are defined about the same reference point. If the reference points are at different locations, the corrections that must be made to the stability derivatives in figures 1 to 3 are shown in figure 4.

Aerodynamic stiffness matrix $[M_4]$

	x_{COL}	z_{COL}	θ_{COL}	δ_{COL}
x_{row}	0	0	$\bar{q} S_W \left(C_{D_{\alpha_R}} + C_{D_{\alpha_E}} - \alpha_1 C_{D_{\hat{u}_R}} - \alpha_1 C_{D_{\hat{u}_E}} - \alpha_1 C_{L_{\alpha_R}} - \alpha_1 C_{L_{\alpha_E}} + \alpha_1^2 C_{L_{\hat{u}_R}} + \alpha_1^2 C_{L_{\hat{u}_E}}\right)$	$\bar{q} S_W \left(C_{D_{\delta_R}} + C_{D_{\delta_E}} - \alpha_1 C_{L_{\delta_R}} - \alpha_1 C_{L_{\delta_E}}\right)$
z_{row}	0	0	$\bar{q} S_W \left(C_{L_{\alpha_R}} - \alpha_1 C_{L_{u_R}} + \alpha_1 C_{D_{\alpha_R}} - \alpha_1^2 C_{D_{\hat{u}_R}}\right)$	$\bar{q} S_W \left(C_{L_{\delta_R}} + \alpha_1 C_{D_{\delta_R}}\right)$
θ_{row}	0	0	$\bar{q} S_W \bar{c} \left(-C_{m_{\alpha_R}} + \alpha_1 C_{m_{\hat{u}_R}}\right)$	$-\bar{q} S_W \bar{c} C_{m_{\delta_R}}$

Aerodynamic damping matrix $[M_5]$

	x_{COL}	z_{COL}	θ_{COL}	δ_{COL}
x_{row}	$\frac{\bar{q} S_W}{U_1} \left(C_{D_{\hat{u}_R}} + C_{D_{\hat{u}_E}} - \alpha_1 C_{L_{\hat{u}_R}} - \alpha_1 C_{L_{\hat{u}_E}}\right)$	$\frac{\bar{q} S_W}{U_1} \left(C_{D_{\alpha_R}} + C_{D_{\alpha_E}} - \alpha_1 C_{L_{\alpha_R}} - \alpha_1 C_{L_{\alpha_E}} - C_{L_{1_R}} - C_{L_{1_E}}\right)$	$\frac{\bar{q} S_W \bar{c}}{2 U_1} \left(C_{D_{\hat{q}_R}} + C_{D_{\hat{q}_E}} + C_{D_{\hat{\dot{\alpha}}_E}} + C_{D_{\hat{\dot{\alpha}}_E}} - \alpha_1 C_{L_{\hat{q}_R}} - \alpha_1 C_{L_{\hat{q}_E}} - \alpha_1 C_{L_{\hat{\dot{\alpha}}_R}} - \alpha_1 C_{L_{\hat{\dot{\alpha}}_E}}\right)$	0
z_{row}	$\frac{\bar{q} S_W}{U_1} \left(C_{L_{\hat{u}_R}} + \alpha_1 C_{D_{\hat{u}_R}}\right)$	$\frac{\bar{q} S_W}{U_1} \left(C_{L_{\alpha_R}} + \alpha_1 C_{D_{\dot{\alpha}_R}}\right)$	$\frac{\bar{q} S_W \bar{c}}{2 U_1} \left(C_{L_{\hat{q}_R}} + C_{L_{\hat{\dot{\alpha}}_R}} + \alpha_1 C_{D_{\hat{q}_R}} + \alpha_1 C_{D_{\hat{\dot{\alpha}}_R}}\right)$	0
θ_{row}	$-\frac{\bar{q} S_W \bar{c}}{U_1} C_{m_{\hat{u}_R}}$	$-\frac{\bar{q} S_W \bar{c}}{U_1} C_{m_{\alpha_R}}$	$\frac{\bar{q} S_W \bar{c}^2}{2 U_1} \left(-C_{m_{\hat{q}_R}} - C_{m_{\hat{\dot{\alpha}}_R}}\right)$	0

x_{COL}, z_{COL}, θ_{COL}, and δ_{COL} are the column locations of the x, z, θ, and δ freedoms.
(Note: There may be more than one control surface freedom.)

The $[M_4]$, $[M_5]$ elements are defined in the inertial axis system.

Figure 1. — Formulation of The Rigid-Body Symmetric Generalized Aerodynamic Stiffness and Damping Matrix Elements Using Stability Derivatives

Aerodynamic stiffness matrix $[M_4]$

	y_{COL}	ϕ_{COL}	ψ_{COL}	δ_{COL}
y_{ROW}	0	$\bar{q} S_W (-C_{L_{1_R}} - C_{L_{1_E}} - \alpha_1 C_{y_{\beta_R}})$	$\bar{q} S_W C_{y_{\beta_R}}$	$-\bar{q} S_W C_{y_{\delta_R}}$
ψ_{ROW}	0	$\bar{q} S_W b (-C_{\ell_{\beta_R}} \cos\alpha_1 + C_{n_{\beta_R}} \sin\alpha_1)\alpha_1$	$\bar{q} S_W b \left(C_{\ell_{\beta_R}} \cos\alpha_1 - C_{n_{\beta_R}} \sin\alpha_1\right)$	$\bar{q} S_W b \left(-C_{\ell_{\delta_R}} \cos\alpha_1 + C_{n_{\delta_R}} \sin\alpha_1\right)$
ϕ_{ROW}	0	$\bar{q} S_W b (-C_{n_{\beta_R}} \cos\alpha_1 - C_{\ell_{\beta_R}} \sin\alpha_1)\alpha_1$	$\bar{q} S_W b \left(C_{n_{\beta_R}} \cos\alpha_1 + C_{\ell_{\beta_R}} \sin\alpha_1\right)$	$\bar{q} S_W b \left(-C_{n_{\delta_R}} \cos\alpha_1 - C_{\ell_{\delta_R}} \sin\alpha_1\right)$

Aerodynamic damping matrix $[M_5]$

	y_{COL}	ϕ_{COL}	ψ_{COL}	δ_{COL}
y_{ROW}	$-\frac{\bar{q} S_W}{U_1} C_{y_{\beta_R}}$	$\frac{\bar{q} S_W b}{2 U_1} \left(-C_{y_{\hat{p}_R}} - \alpha_1 C_{y_{\hat{\beta}_R}}\right)$	$\frac{\bar{q} S_W b}{2 U_1} \left(-C_{y_{\hat{r}_R}} + C_{y_{\hat{\beta}_R}}\right)$	0
ϕ_{ROW}	$\frac{\bar{q} S_W b}{U_1} \left(-C_{\ell_{\beta_R}} \cos\alpha_1 + C_{n_{\beta_R}} \sin\alpha_1\right)$	$\frac{\bar{q} S_W b^2}{2 U_1} \left(-C_{\ell_{\hat{p}_R}} \cos\alpha_1 + C_{n_{\hat{p}_R}} \sin\alpha_1 - C_{\ell_{\hat{\beta}_R}} (\cos\alpha_1)\alpha_1 + C_{n_{\hat{\beta}_R}} (\sin\alpha_1)\alpha_1\right)$	$\frac{\bar{q} S_W b^2}{2 U_1} \left(-C_{\ell_{\hat{r}_R}} \cos\alpha_1 + C_{n_{\hat{r}_R}} \sin\alpha_1 + C_{\ell_{\hat{\beta}_R}} \cos\alpha_1 - C_{n_{\hat{\beta}_R}} \sin\alpha_1\right)$	0
ψ_{ROW}	$\frac{\bar{q} S_W b}{U_1} \left(-C_{n_{\beta_R}} \cos\alpha_1 - C_{\ell_{\beta_R}} \sin\alpha_1\right)$	$\frac{\bar{q} S_W b^2}{2 U_1} \left(-C_{n_{\hat{p}_R}} \cos\alpha_1 - C_{\ell_{\hat{p}_R}} \sin\alpha_1 - C_{n_{\hat{\beta}_R}} (\cos\alpha_1)\alpha_1 - C_{\ell_{\hat{\beta}_R}} (\sin\alpha_1)\alpha_1\right)$	$\frac{\bar{q} S_W b^2}{2 U_1} \left(-C_{n_{\hat{r}_R}} \cos\alpha_1 - C_{\ell_{\hat{r}_R}} \sin\alpha_1 + C_{n_{\hat{\beta}_R}} \cos\alpha_1 + C_{\ell_{\hat{\beta}_R}} \sin\alpha_1\right)$	0

The $[M_4]$ and $[M_5]$ elements are defined in the inertial axis system

Figure 2. — Formulation of the Rigid-Body Antisymmetric Generalized Aerodynamic Stiffness and Damping Matrix Elements Using Stability Derivatives

(a) Symmetric

Gust forcing column $[C_3]$

x ROW	$\frac{\bar{q}\, S_W}{U_1}\left(-C_{D_{\alpha_R}} - C_{D_{\alpha_E}} + \alpha_1 C_{L_{\alpha_R}} + \alpha_1 C_{L_{\alpha_E}} + C_{L_{1_R}} + C_{L_{1_E}}\right)$
z ROW	$\frac{\bar{q}\, S_W}{U_1}\left(-C_{L_{\alpha_R}} - \alpha_1 C_{D_{\alpha_R}}\right)$
θ ROW	$\frac{\bar{q}\, S_W\, \bar{c}}{U_1}\, C_{m_{\alpha_R}}$

This is the z_{COL} of the M_5 matrix with the sign changed. (Note: Due to the sign convention adopted in EOM, only the symmetric case requires a sign change.)

(b) Antisymmetric

Gust forcing column $[C_3]$

y ROW	$-\frac{\bar{q}\, S_W}{U_1}\, C_{Y_{\beta_R}}$
ϕ ROW	$\frac{\bar{q}\, S_W\, b}{U_1}\left(-C_{\ell_{\beta_R}} \cos \alpha_1 + C_{n_{\beta_R}} \sin \alpha_1\right)$
ψ ROW	$\frac{\bar{q}\, S_W\, b}{U_1}\left(-C_{n_{\beta_R}} \cos \alpha_1 - C_{\ell_{\beta_R}} \sin \alpha_1\right)$

This is the y_{COL} of the M_5 matrix.

Figure 3. – Formulation of the Rigid-Body Gust Excitation Matrix Elements Using Stability Derivatives

(a) Symmetric

$$C_{m_{\hat{u}_R}} = C_{m_{\hat{u}_{R_{REF}}}} + \frac{\Delta x}{\bar{c}} C_{L_{\hat{u}_R}} - \frac{\Delta z}{\bar{c}} C_{D_{\hat{u}_R}}$$

$$C_{m_{\alpha_R}} = C_{m_{\alpha_{R_{REF}}}} + \frac{\Delta x}{\bar{c}} C_{L_{\alpha_R}} - \frac{\Delta z}{\bar{c}} C_{D_{\alpha_R}}$$

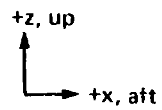
$$C_{m_{\hat{q}_R}} = C_{m_{\hat{q}_{R_{REF}}}} + \frac{\Delta x}{\bar{c}} C_{L_{\hat{q}_R}} - \frac{\Delta z}{\bar{c}} C_{D_{\hat{q}_R}}$$

$$C_{m_{\dot{\alpha}_R}} = C_{m_{\hat{\dot{\alpha}}_{R_{REF}}}} + \frac{\Delta x}{\bar{c}} C_{L_{\hat{\dot{\alpha}}_R}} - \frac{\Delta z}{\bar{c}} C_{D_{\hat{\dot{\alpha}}_R}}$$

$$C_{m_{\delta_R}} = C_{m_{\delta_{R_{REF}}}} + \frac{\Delta x}{\bar{c}} C_{L_{\delta_R}} - \frac{\Delta z}{\bar{c}} C_{D_{\delta_R}}$$

Where $\Delta x = (x_{REF} - x_{AERO\ REF})$

$\Delta z = (z_{REF} - z_{AERO\ REF})$



Aero ref = Reference point about which the aerodynamic derivatives are calculated.

Ref = Reference point about which the rigid-body motion is defined in the analysis.

Figure 4. – Stability Derivative Corrections for a Different Aero Reference Point

(b) Antisymmetric

$$C_{\ell_{\beta_R}} = C_{\ell_{\beta_{R_{REF}}}} - \frac{\Delta z}{b} C_{y_{\beta_R}}$$

$$C_{\ell_{\hat{\beta}_R}} = C_{\ell_{\hat{\beta}_{R_{REF}}}} - \frac{\Delta z}{b} C_{y_{\hat{\beta}_R}}$$

$$C_{\ell_{\hat{p}_R}} = C_{\ell_{\hat{p}_{R_{REF}}}} - \frac{\Delta z}{b} C_{y_{\hat{p}_R}}$$

$$C_{\ell_{\hat{r}_R}} = C_{\ell_{\hat{r}_{R_{REF}}}} - \frac{\Delta z}{b} C_{y_{\hat{r}_R}}$$

$$C_{\ell_{\delta_R}} = C_{\ell_{\delta_{R_{REF}}}} - \frac{\Delta z}{b} C_{y_{\delta_R}}$$

and

$$C_{n_{\beta_R}} = C_{n_{\beta_{R_{REF}}}} + \frac{\Delta x}{b} C_{y_{\beta_R}}$$

$$C_{n_{\hat{\beta}_R}} = C_{n_{\hat{\beta}_{R_{REF}}}} + \frac{\Delta x}{b} C_{y_{\hat{\beta}_R}}$$

$$C_{n_{\hat{p}_R}} = C_{n_{\hat{p}_{R_{REF}}}} + \frac{\Delta x}{b} C_{y_{\hat{p}_R}}$$

$$C_{n_{\hat{r}_R}} = C_{n_{\hat{r}_{R_{REF}}}} + \frac{\Delta x}{b} C_{y_{\hat{r}_R}}$$

$$C_{n_{\delta_R}} = C_{n_{\delta_{R_{REF}}}} + \frac{\Delta x}{b} C_{y_{\delta_R}}$$

Figure 4. – (Concluded)

4.2 ACTIVE CONTROL SYSTEM DEFINITION AND SENSOR EQUATIONS

If the effects of an active control system are to be represented in the equations of motion of an airplane (eq. 1), it is necessary to define the active control system as a number of linear second order or less differential equations. As an example of this procedure, a set of linear differential equations are developed using the sample control system in figure 5. This control system is only an example used for the purpose of illustration. The user is free to use any type of control system as long as its mathematical description can be reduced to a set of linear second order or less differential equations.

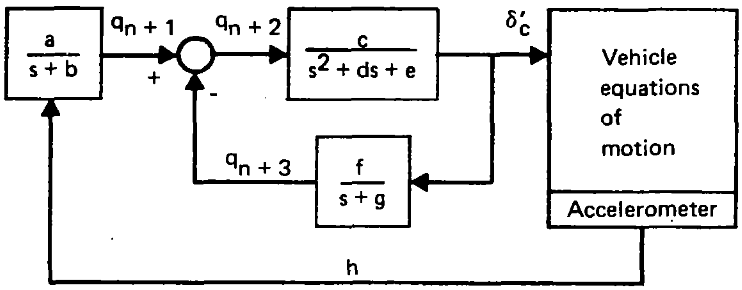


Figure 5. — Sample Control System

where:

q_1 to q_n = Vehicle degrees of freedom (generalized coordinates)

q_n = Control surface rotation (δ_c)

δ_c' = Amount of control surface rotation commanded by the control system

h = Acceleration at a particular sensor location

q_{n+1} to
q_{n+3} = Dummy degrees of freedom

Throughout this section, the control system equations will be developed in the Laplace domain (s-plane); therefore, all generalized coordinates and control surface rotations will be functions of the Laplace variable, **s**.

In the example of figure 5, the equations of motion were derived using **n** degrees of freedom with the n^{th} generalized coordinate being the actual control surface rotation, δ_c. However, this is not a requirement of the program. The control surface rotation may occupy any position in the generalized coordinate array. It is also assumed, for the sake of simplicity, that the amount of rotation experienced by the control surface, δ_c, will be equal to the amount of rotation commanded by the control system, δ_c'; that is, the control system is perfect. This is usually not the case. In reality the physical properties of a control system (e.g., the deflection of backup structure, maximum actuator force available, etc.) coupled with the aerodynamic forces on the control surface will result in a difference between the amount of control rotation commanded and the actual amount experienced. In such instances, the user must supply the appropriate equations that will describe this difference.

In the following equations, q_1 to q_n represent the airplane's elastic and rigid body degrees of freedom. The quantity **h** will be an acceleration sensed on the vehicle and used as feedback input by the control system. In general, the quantity **h** may be displacement, velocity, or acceleration. q_{n+1} to q_{n+3} are dummy degrees of freedom used to keep the control system equations to a set of second order or less differential equations.

Working through the block diagram shown in figure 5, the set of differential equations describing the sample control system is derived as follows:

a) Based on the assumption of a perfect control system

$$\delta c = \delta c'$$

or

$$\delta c - \delta c' = 0 \tag{5}$$

b) From the outer loop shown in figure 5

$$q_{n+1} = \frac{a}{s+b} h$$

or

$$(s+b)\, q_{n+1} - ah = 0 \tag{6}$$

c) The relationship between the acceleration, **h**, and the generalized coordinates excluding the dummy coordinates is

$$\lfloor \phi_h \rfloor \begin{Bmatrix} \ddot{q}_1 \\ \vdots \\ \ddot{q}_n \end{Bmatrix} = s^2 \lfloor \phi_h \rfloor \begin{Bmatrix} q_1 \\ \vdots \\ q_n \end{Bmatrix} = h$$

or

$$s^2 \lfloor \phi_h \rfloor \begin{Bmatrix} q_1 \\ \vdots \\ q_n \end{Bmatrix} - h = 0 \tag{7}$$

The mode shapes $\lfloor \phi_h \rfloor$ are the modal deflections at the particular sensor location and they can be obtained from the program LOADS(L218). Equation 7 may be considered as the sensor equation.

d) From the summation point

$$q_{n+2} = q_{n+1} - q_{n+3}$$

or

$$q_{n+2} - q_{n+1} + q_{n+3} = 0 \tag{8}$$

e) The forward path gives

$$\delta c' = \frac{c}{s^2 + ds + e}\, q_{n+2}$$

or

$$(s^2 + ds + e)\,\delta c' - c q_{n+2} = 0 \tag{9}$$

f) Finally, from the inner feedback loop

$$q_{n+3} = \frac{f}{s + g}\, \delta c'$$

or

$$(s + g)\, q_{n+3} - (f)\delta c' = 0 \tag{10}$$

The coefficients for equations (5) through (10) relating the addition degrees of freedom (q_{n+1}, q_{n+2}, q_{n+3}, $\delta_c{}'$ and h) to the unaugmented vehicle degrees of freedom (q_1 to q_n) must be added to the unaugmented equations of motion matrix coefficients.

Taking the Laplace transform of the equations of motion (eq. 1), the unaugmented equations of motion matrix coefficients of $[M_1]$, $[M_2]$, and $[M_3]$ represented by square matrices of size (n X n), can be written as coefficients of **s**.

$$\left[M_{1UA}\right]\begin{Bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ \delta c \end{Bmatrix} + s\left[M_{2UA}\right]\begin{Bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ \delta c \end{Bmatrix} + s^2\left[M_{3UA}\right]\begin{Bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ \delta c \end{Bmatrix} \tag{11}$$

The equations of motion are now expanded by five degrees of freedom with the coefficients of equations (5) through (10) placed in the appropriate matrix locations as shown in the following:

$$\left[\begin{array}{c:ccccc} M_{1UA} & & & 0 & & \\ 1 & & & & & -1 \\ \hdashline & b & 0 & 0 & -a & 0 \\ & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -c & 0 & 0 & e \\ & 0 & 0 & 0 & -1 & 0 \\ & 0 & 0 & g & 0 & -f \end{array}\right]\begin{Bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ \delta c \\ q_{n+1} \\ q_{n+2} \\ q_{n+3} \\ h \\ \delta c' \end{Bmatrix} + s\left[\begin{array}{c:ccccc} M_{2UA} & & & 0 & & \\ & & & & & \\ \hdashline & 1 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d \\ & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 1 & 0 & 0 \end{array}\right]\begin{Bmatrix} q_1 \\ \vdots \\ q_{n-1} \\ \delta c \\ q_{n+1} \\ q_{n+2} \\ q_{n+3} \\ h \\ \delta c' \end{Bmatrix}$$

$$
+ s^2 \left[\begin{array}{c:ccccc} M_{3UA} & & & 0 & & \\ \hdashline & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 1 \\ \lfloor \phi_h \rfloor & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{Bmatrix} q_1 \\ \cdot \\ \cdot \\ q_{n-1} \\ \delta c \\ q_{n+1} \\ q_{n+2} \\ q_{n+3} \\ h \\ \delta c' \end{Bmatrix} \qquad (12)
$$

Each added row in equation 12 corresponds to one of the equations 5 through 10.

The inclusion of the control system equations, as illustrated in this example, is accomplished in EQMOD by the user specifying on cards the coefficients and their row and column locations in the augmented matrices. The aerodynamic related matrices, $[M_4]$, $[M_5]$ and either $\{C_3\}$ or $[\tilde{\phi}]$, are automatically augmented by rows and columns of zeroes. No aerodynamic forces and moments result from adding the dummy degrees of freedom.

It is important to note that the sensor equation for this sample control system used acceleration as a feedback quantity. Therefore, the mode shape matrix, ϕ_n was placed in the augmented M_3 matrix. In the case of displacement or velocity being the feedback variable, the modal data would have been placed in the augmented M_1 or M_2, respectively. These sensor coefficients (mode shapes) may be either calculated in L219 (LOADS) (ref. 4) and read directly into L219 (EQMOD), or they can be input manually into the program by cards.

Also with regard to the sensor equation (7), the variable **h** is automatically included by EQMOD in the augmented generalized coordinate array. In defining the sensor data, the user need only specify the matrix (M_1, M_2, M_3) and the row at the matrix where the mode shapes are to be placed and EQMOD automatically places a -1 in the proper column of M_1. In the sample problem, the mode shapes were placed in the $(n+4)^{th}$ row of the augmented M_3 matrix (eq. 12), therefore the -1 was placed in the $(n+4)^{th}$ row and $(n+4)^{th}$ column of the augmented M_1 matrix.

This example used only one sensor **h**. EQMOD can accept more than one sensor and more than one type of sensor. However, the method used for more sensors is identical to that used in this example.

4.3 MATRIX MODIFICATION BY SCALAR MULTIPLICATION, REPLACEMENT OR INCREMENTATION OF MATRIX ELEMENTS.

EQMOD offers the capability for modifying all the coefficient matrices in the equations of motion (1) and (2) load equations (3) and (4). The matrix coefficients may be modified by multiplying the entire matrix by scalar factors or by replacing or incrementing individual elements within the matrices. The following is an example of matrix modification by scalar multiplication.

In the generation of the $[M_4]$ and $[\overline{M}_4]$ matrices, the dynamic pressure is embedded in the coefficients and thus $[M_4]$ and $[\overline{M}_4]$ are functions of the dynamic pressure, $\overline{q}$.

$$[M_4], [\overline{M}_4] = f(q) \tag{13}$$

Similarly, the $[M_5]$, $[\overline{M}_5]$, $\{C_3\}$, or $[\Phi]$ and $\{\overline{C}_3\}$ or $[\overline{\Phi}]$ matrix coefficients are functions of $\overline{q}$ and the reciprocal of the freestream velocity, V_T.

$$[M_5], [\overline{M}_5], a \{C_3\}, \{\overline{C}_3\} = f(\overline{q}, \frac{1}{V_T}) \tag{14}$$

If a parameter study is desired varying only the dynamic pressure and freestream velocity, but keeping everything else constant including the Mach number, the $[M_4]$ and $[\overline{M}_4]$ matrices can be multiplied by a scalar which is the ratio of the new dynamic pressure to the original dynamic pressure.

$$[M_4]_{new} = \frac{\overline{q}_{new}}{\overline{q}} [M_4]$$

$$[\overline{M}_4]_{new} = \frac{\overline{q}_{new}}{\overline{q}} [\overline{M}_4] \tag{15}$$

$[M_5]$, $[\overline{M}_5]$, $\{C_3\}$ or $[\Phi]$, and $\{\overline{C}_3\}$ or $[\overline{\Phi}]$ matrices can be multiplied by a scalar consisting of this ratio times the ratio of at the original velocity to the new velocity

$$[M_5]_{new} = \frac{V_T}{V_{T_{new}}} \frac{\overline{q}_{new}}{\overline{q}} [M_5] \tag{16}$$

and so on.

In performing the matrix modifications in EQMOD, the values of the scalar multipliers, the replacement elements, and the element increments must be calculated by the user and input via cards. EQMOD dos not calculate those values internally.

4.4 FORMATION OF EQUATION OF MOTION CHARACTERISTIC EQUATION WITH WAGNER INDICIAL LIFT GROWTH FUNCTION

A method of obtaining the stability of a system is to calculate the roots of the system's characteristic equation. The Linear System Analysis Program (QR) (ref. 6) has the capability of calculating the characteristic equation of a system from the Laplace transform of the equations of motion (1) and determining the roots of that equation. It is the function of EQMOD to formulate a set of matrix coefficients which represent the Laplace transform of equation (1). The following is the theory to form this set of matrix coefficients that includes indicial lift growth functions applied to the equations of motion generalized aerodynamic coefficient matrices.

In the time domain, the Wagner indicial lift growth function may be approximated as

$$\Phi(t) = 1 - a_1 e^{-\alpha_1 t} - b_1 e^{-\beta_1 t} \tag{17}$$

The Laplace transform of equation (17) is

$$\Phi(s) = \frac{1}{s} - \frac{a_1}{s + \alpha_1} - \frac{b_1}{s + \beta_1} \tag{18}$$

In equation (1), the Wagner indicial lift growth function is convoluted with $\dot{q}$ and $\ddot{q}$. Using the relationship for the Laplace transform of the convolution integral (Duhamel's formula):

$$\mathscr{L}\left[\int_o^t f(t - \lambda)\, g(\lambda)\, d\lambda\right] = \mathscr{L}\left[f(t) * g(t)\right] = \mathscr{L}[f(t)]\; \mathscr{L}[g(t)] \tag{19}$$

the left side of equation (1) in the Laplace domain becomes:

$$\Big[[M_1] + s[M_2] + s^2\,[M_3] + s[M_4]\,\frac{1}{s}\Big(1 - \frac{a_1 s}{s + \alpha_1} - \frac{b_1 s}{s + \beta_1}\Big)$$

$$+ s^2\,[M_5]\,\frac{1}{s}\Big(1 - \frac{a_1 s}{s + \alpha_1} - \frac{b_1 s}{s + \beta_1}\Big)\Big] \;\{\mathscr{L}(q)\} = 0 \tag{20}$$

Expanding and collecting terms gives:

$$\Big[s^4\,[M_3] + s^3\,\Big[[M_2] + [M_5] + (\alpha_1 + \beta_1)[M_3] - (a_1 + b_1)[M_5]\Big]$$

$$+ s^2\Big[[M_1] + [M_4] + (\alpha_1 + \beta_1)\Big[[M_2] + [M_5]\Big] + \alpha_1\;\beta_1[M_3]$$

$$- (a + b)[M_4] - (a_1\beta_1 + b_1\alpha_1)[M_5]\Big]$$

$$+ s\Big[(\alpha_1 + \beta_1)\Big[[M_1] + [M_4]\Big] + \alpha_1\beta_1\Big[[M_2] + [M_5]\Big]$$

$$- (a_1\beta_1 + b_1\alpha_1)[M_4]\Big] + \alpha_1\beta_1\Big[[M_1] + [M_4]\Big]\Big]\{\mathscr{L}(q)\} = 0 \tag{21}$$

If indicial lift growth is not included, that is:

$$\Phi\ (t) = 1$$

or

$$\Phi\ (s) = \frac{1}{s} \tag{22}$$

and consequently a_1 and b_1 from equation (18) are zero, equation (21) will simplify into only a second power of **s** equation.

The characteristic equation of the system is obtained by taking the determinant of equation (21) and setting it equal to zero. It should be noted that due to the inclusion of the Wagner function, the order of the characteristic equation is increased and additional roots will be calculated. These additional roots do not represent additional modes of the system. The number of additional roots is a function of the order of the set of polynomial equations shown in equation (21) and of the number of degrees of freedom in the system. The order of the set of polynomials is a function of the number of terms used in the approximation of the Wagner function.

The function of EQMOD is to form the matrix coefficients of **s** shown in equation (21).

4.5 TRANSFORMATION FROM INERTIAL AXES TO BODY-FIXED AXES

The matrices for equations (1) and (3) must be generated in the inertial axis system for straight and level flight. If the user prefers to work in the body axis system, a transformation from the inertia to body-fixed axis system may be desirable and can be accomplished.

Presented in this section is a summary of the matrix changes that are made by EQMOD to the coefficients of equations (1) and (3) in order to convert from inertia axes to body-fixed axes. A full theoretical development of the transformation is given in appendix A.

Basically, the effect of the transformation is to redefine the generalized coorinates that describe the rigid body motions of the aircraft in equations (1) and (3). All other coordinates (elastic and control deflections) are not affected. In the inertia axes, which are fixed in space, the motion of the aircraft relative to these axes is described by the velocity components in the direction of the inertia axes. In the body-fixed axes, however, the motion is described by the velocity relative to the fixed inertia axes but in the direction of the moving axes. The relationships between the velocity components in the inertia axes and the body-fixed axes are shown in table 1.

U_1 and W_1 are the reference (in this case lg) values of velocity defined in the reference axis system which is fixed. U_1 will be referred to as the airplane forward velocity, V_T. For small angles, the reference (lg) angle of attack can be defined as

$$\alpha_1 \approx \tan \alpha_1 = \frac{W_1}{V_T} \tag{23}$$

Table 1. — Relationships Between Velocity Components in the Inertia Axes and the Body-Fixed Axes

Analysis	Inertia axes	Body-fixed axes
Symmetric	$\dot{x}'$	$u + W_1\theta'$
	$\dot{z}'$	$w - U_1\theta'$
	$\dot{\theta}'$	q
Antisymmetric	$\dot{y}'$	$v + U_1\psi' - W_1\phi'$
	$\dot{\phi}'$	p
	$\dot{\psi}'$	r

The effects of the transformation on the meaning of the generalized coordinates and the changes in the various matrices will be examined for the symmetric analysis first and the antisymmetric analysis second.

Symmetric Analysis

In the symmetric analysis, the generalized coordinates can be interpreted as:

In the inertia axis system

$$\{q\} = \begin{Bmatrix} x' \\ z' \\ \theta' \\ q_e \\ \delta_c \end{Bmatrix} \qquad \{\dot{q}\} = \begin{Bmatrix} \dot{x}' \\ \dot{z}' \\ \dot{\theta}' \\ \dot{q}_e \\ \dot{\delta}_c \end{Bmatrix} \qquad \{\ddot{q}\} = \begin{Bmatrix} \ddot{x}' \\ \ddot{z}' \\ \ddot{\theta}' \\ \ddot{q}_e \\ \ddot{\delta}_c \end{Bmatrix} \tag{24}$$

and in the body-fixed axis system

$$\{q\} = \begin{Bmatrix} x \\ y \\ \theta \\ q_e \\ \delta_c \end{Bmatrix} \qquad \{\dot{q}\} = \begin{Bmatrix} u \\ w \\ q \\ q_e \\ \delta_c \end{Bmatrix} \qquad \{\ddot{q}\} = \begin{Bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \ddot{q}_e \\ \ddot{\delta}_c \end{Bmatrix} \tag{25}$$

Note the elastic coordinates, q_e , and the control surface coordinates, δ_c, remain unchanged from one system to the next.

To transform equations (1) and (3) to body-fixed axes, the rigid body generalized coordinate velocity and acceleration matrices in the inertia axes are replaced with the expressions given in table 1 and with derivatives of these expressions. The resulting terms are then regrouped into coefficients of generalized coordinate displacements, velocity, and acceleration. In doing so, the transformation from inertia to body-fixed axis system requires the following changes to the coefficient matrices $[M_1]$, $[M_2]$, $[M_4]$, $[\overline{M}_1]$, $[\overline{M}_2]$, and $[\overline{M}_4]$.

The only column changed in these matrices is the θ:

$$\begin{array}{ll} & \theta_{col} \text{ to} \\ \text{in}[M_1]: & M_{1_{\theta_{col}}} - V_T\left(M_{2_{z_{col}}} - \alpha_1 M_{2_{x_{col}}}\right) \\ \text{in}[M_2]: & M_{2_{\theta_{col}}} - V_T\left(M_{3_{z_{col}}} - \alpha_1 M_{3_{x_{col}}}\right) \\ \text{in}[M_4]: & M_{4_{\theta_{col}}} - V_T\left(M_{5_{z_{col}}} - \alpha_1 M_{5_{x_{col}}}\right) \end{array} \tag{26}$$

$M_{n_{a_{col}}}$ = the a column of the original (inertia axis) matrix of the n^{th} matrix

$[\overline{M}_1]$, $[\overline{M}_2]$, and $[\overline{M}_4]$ are changed in the same manner. The transformation does not affect the $[M_3]$, $[\overline{M}_3]$, $[M_5]$, or $[\overline{M}_5]$ matrices.

Antisymmetric Analysis

Similarly, in the anti-symmetric analysis, the generalized coordinates can be interpreted as:

In the inertia axis system:

$$\{q\} = \begin{Bmatrix} y' \\ \phi' \\ \psi' \\ q_e \\ \delta_c \end{Bmatrix}, \quad \{\dot{q}\} = \begin{Bmatrix} \dot{y}' \\ \dot{\phi}' \\ \dot{\psi}' \\ \dot{q}_e \\ \dot{\delta}_c \end{Bmatrix}, \{\ddot{q}\} = \begin{Bmatrix} \ddot{y}' \\ \ddot{\phi}' \\ \ddot{\psi}' \\ \ddot{q}_e \\ \ddot{\delta}_c \end{Bmatrix} \tag{27}$$

And in the body-fixed axis system:

$$\{q\} = \begin{Bmatrix} y \\ \phi \\ \psi \\ q_e \\ \delta_c \end{Bmatrix}, \quad \{\dot{q}\} = \begin{Bmatrix} v \\ p \\ r \\ \dot{q}_e \\ \dot{\delta}_c \end{Bmatrix}, \{\ddot{q}\} = \begin{Bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \ddot{q}_e \\ \ddot{\delta}_c \end{Bmatrix} \tag{28}$$

Performing a similar substitution as with the symmetric analysis — but with the antisymmetric variables in table 1 — and regrouping, the following changes to the coefficient matrices are required:

$$\begin{array}{l|l|l|}
 & \phi_{col} \text{ to} & \psi_{col} \text{ to} \\
\text{in } [M_1]: & M_{1_{\phi\,col}} - V_T\,\alpha_1 M_{2_{y\,col}} & M_{1_{\psi\,col}} + V_T M_{2_{y\,col}} \\
\text{in } [M_2]: & M_{2_{\phi\,col}} - V_T\,\alpha_1 M_{3_{y\,col}} & M_{2_{\psi\,col}} + V_T M_{3_{y\,col}} \\
\text{in } [M_4]: & M_{4_{\phi\,col}} - V_T\,\alpha_1 M_{5_{y\,col}} & M_{4_{\psi\,col}} + V_T M_{5_{y\,col}}
\end{array} \tag{29}$$

$[\overline{M}_1]$, $[\overline{M}_2]$, and $[\overline{M}_4]$ are changed in the same manner. Again, $[M_3]$, $[M_3]$, $[M_5]$, and $[\overline{M}_5]$ are not affected by the transformation. EQMOD performs all matrix manipulations needed to complete the transformation. The user needs only define the column locations of the rigid body motions, the 1g angle of attack and the airplane's forward velocity, V_T.

5.0 PROGRAM STRUCTURE AND DESCRIPTION

L219 (EQMOD) has been constructed as an overlay system. Figure 6 shows the overlay structure and the data input to and output from each overlay. The overlays are:

Main overlay (L219,0,0)	L219vc
Primary overlay (L219,1,0)	RDCRDS
Secondary overlay (L219,1,1)	RDEOM
Secondary overlay (L219,1,2)	RDLOD
Secondary overlay (L219,1,3)	RDQR
Primary overlay (L219,2,0)	EOMMOD
Primary overlay (L219,3,0)	LODMOD
Primary overlay (L219,4,0)	QRMOD

The main overlay, L219vc ("v" and "c" are version and correction identifiers), is a small program that calls into execution the primary overlays required to perform the operations requested by the user via card input data.

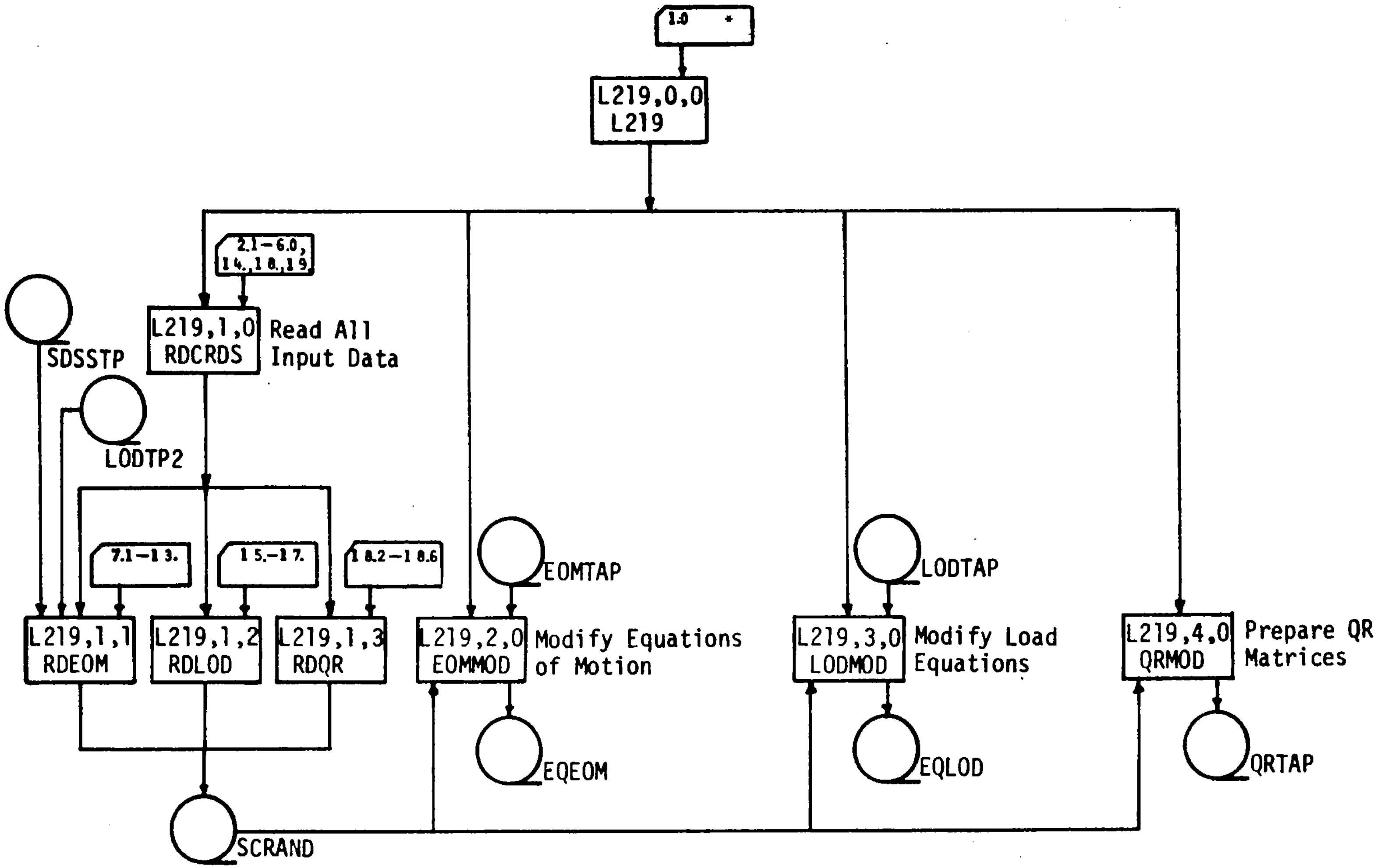
The first overlay called into execution by L219 is always the 1,0 primary overlay RDCRDS. RDCRDS reads and interprets all card input data, determines the execution options, and writes the edited input data onto the random scratch file SCRAND. RDCRDS calls three secondary overlays to process special sections of the input data:

(L219,1,1) RDEOM reads instructions from cards directing the modification of equations of motion matrices. FLEXSTAB (with the DYLOFLEX modification, ref. 7) stability derivatives are read from the file SDSSTP, if required. Also, sensor equations will be read from the file LODTP2, if requested.

(L219,1,2) RDLOD reads instructions from cards directing the modification of load equation matrices.

(L219,1,3) RDQR reads instructions from cards directing the preparation of matrices for the program QR.

The remaining primary overlays (2,0, 3,0, and 4,0) will be executed only if requested by the user. All primary overlays read input instructions from SCRAND.



*Numbers refer to the card sets or cards which are used for input by each overlay.

Figure 6. – L219 (EQMOD) Overlay Structure and Input/Output Files

Overlay 2,0 (EOMMOD) modifies the equations of motion matrices read from the file EOMTAP. The modifications are made in the following order (all are optional):

- Calculate stability derivatives and store over the proper matrix elements.
- Scalar-multiply matrices.
- Add sensor equations.
- Replace and increment elements.
- Perform the inertia to body-axis transformation.

The resulting matrices are written onto the file EQEOM specified by card input in Overlay 1,0 for use in the Random Harmonic Analysis program, L221 (ref. 8).

Overlay 3,0 (LODMOD) modifies the load equation matrices read from the file LODTAP. The changes are made in the following order.

- Scalar-multiply matrices.
- Replace and increment elements.
- Perform inertia to body-axis transformation.

The resulting matrices are written onto the file EQLOD specified by card input in Overlay 1,0 for use in the Random Harmonic Analysis program, L221, (ref. 8).

Overlay 4,0 (QRMOD) prepares matrices for the program QR. The necessary equations of motion and load equation matrices are used to form the matrices for rooting and for a time history solution. The matrices are written onto the file QRTAP as specified on card input in Overlay 1,0.

For a more complete description of the L219 (EQMOD) program structure see volume II of this document.

6.0 COMPUTER PROGRAM USAGE

The program was designed for use on the CDC 6600. The machine requirements to execute L219 (EQMOD) are:

Card reader	To read control cards and card input data.
Printer	To print standard output information, optional intermediate results and diagnostic messages.
Disk storage	All magnetic files not specifically defined as magnetic tapes are assumed to be disk files used for input, output, and temporary file storage.
Tape drive	For "permanent" storage of data; magnetic files are copied to and from magnetic tapes with control cards before and after program execution.

The program L219 (EQMOD) is written in FORTRAN and may be complied with either the RUN or FTN compiler. L219 may be executed on either the KRONOS 2.1 or NOS operating system.

6.1 CONTROL CARDS

The following list is a typical set of control cards used to execute L219 (EQMOD) using the absolute binaries from the program's master tape.

Job card

Account card

•

•

•

```
REQUEST(MASTER,F=I,LB=KL,VSN=66XXXX)     Retrieve the
REWIND(MASTER)                           program from its
SKIPF(MASTER)                            Master tape
COPYBF(MASTER,L219)
RETURN(MASTER)
```

•

•

•

•

•
•
•
L219.
•
•
•
EXIT.

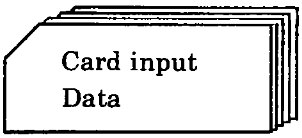
Prepare optional input data files

Excecute L219 (EQMOD)

Save optional output data files

DMP(0,field length)

--- End-of-record



--- End-of-file

6.2 RESOURCE ESTIMATES

The computer resources used (core requirements, tapes, printed output, time, etc.) are a function of the problem size and program options used.

FIELD LENGTH

The field length required by L219 (EQMOD) is dependent upon the problem size and the program module(s) used. Core must be requested based on the largest amount of core needed for any one module to be run, that is:

$$\text{Field length} \quad = \quad \text{Maximum required for} \quad \left\{ \begin{array}{c} \text{RDCRDS} \\ \text{EOMMOD} \\ \text{LODMOD} \\ \text{QRMOD} \end{array} \right\}$$

For each module, the core requirements are determined from the following formulas

RDCRDS $\quad 110{,}000_8$ + NDOF*NDOF

EOMMOD $\quad 71{,}000_8$ + 3*NDOF*NDOF

LODMOD $\quad 66{,}000_8$ + 2*NODF*NDOF + NDOF*NLD

QRMOD $\quad 67{,}000_8$ + 2*NDOF*NDOF

where: NDOF = number degrees of freedom

NLD = number of loads

Time Estimate

The time estimate is dependent upon the problem size. However, the average time to run most average jobs should be less than 25 seconds.

Printed Output

The maximum number of lines of printed output has been limited to 40 000, which should be enough for any L219 (EQMOD) program execution. The average line count is about 1000 lines. If output line limit is exceeded, use the following control card to execute the program:

L219(PL = limit)

where "limit" is the approximate number of lines required to execute this program.

6.3 CARD INPUT DATA

A detailed description of the card input data needed to execute EQMOD is given in sections 6.3.1 through 6.3.4. A summary of the card input data is given in section 6.3.5. The summary is a quick reference for the necessary card input and is included for use only after familiarity with the program has been obtained.

The task(s) performed by L219 (EQMOD) are broken into three subtasks, each with its own section of code known as a primary overlay. The entire set of primary overlays is driven by a small program (main overlay) named L219vc.

L219vc reads program directive cards to:

- Assure that the data being read is intended for L219 (EQMOD).
- Determine which section of code (primary overlay) of L219 is to be executed next.
- Determine what data and results are to be printed.
- Determine input and output magnetic file names.
- Determine total problem size.

The order in which the input cards are read is shown in figure 7.

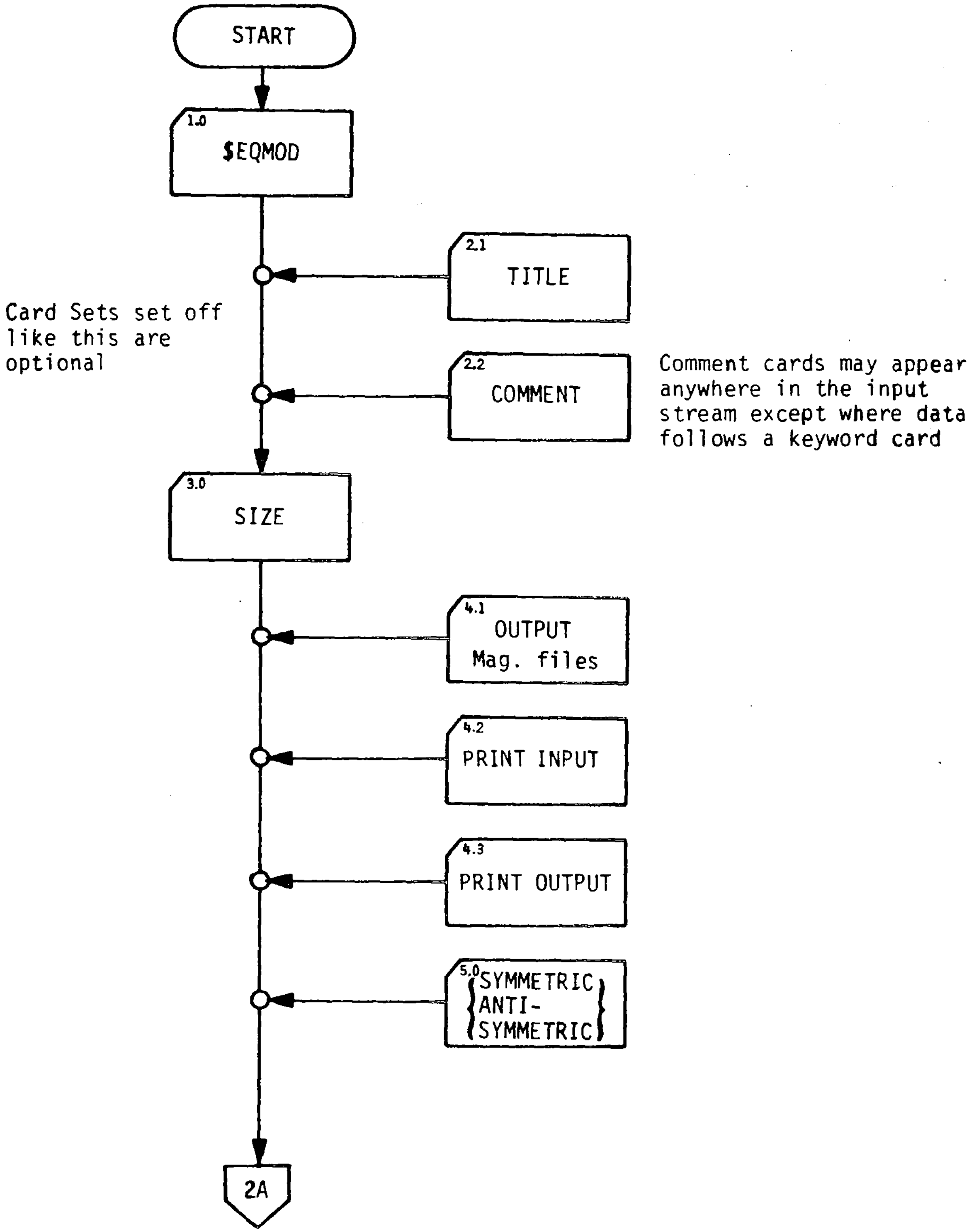


Figure 7. – Flow of Card Input Data

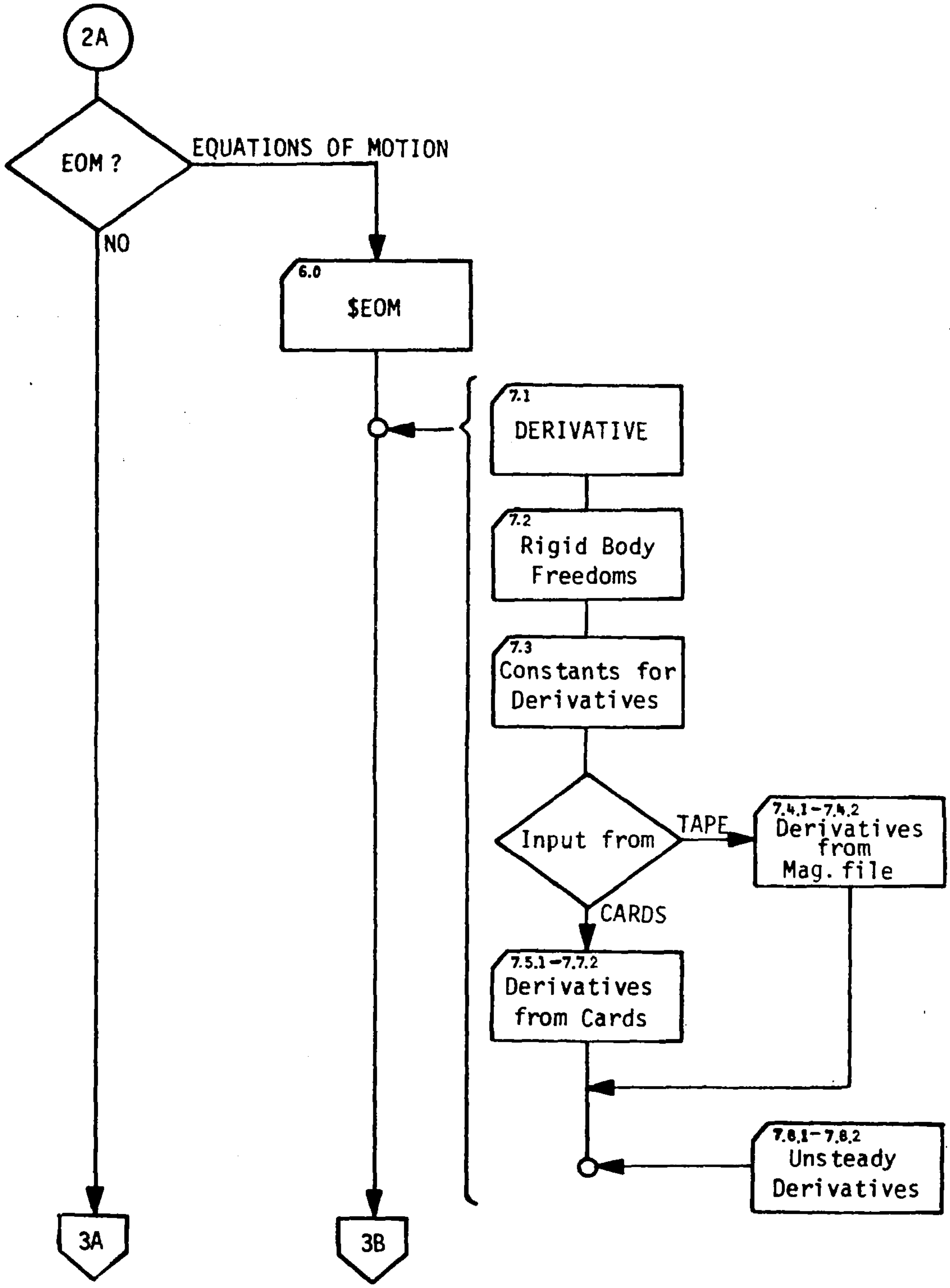


Figure 7. – (Continued)

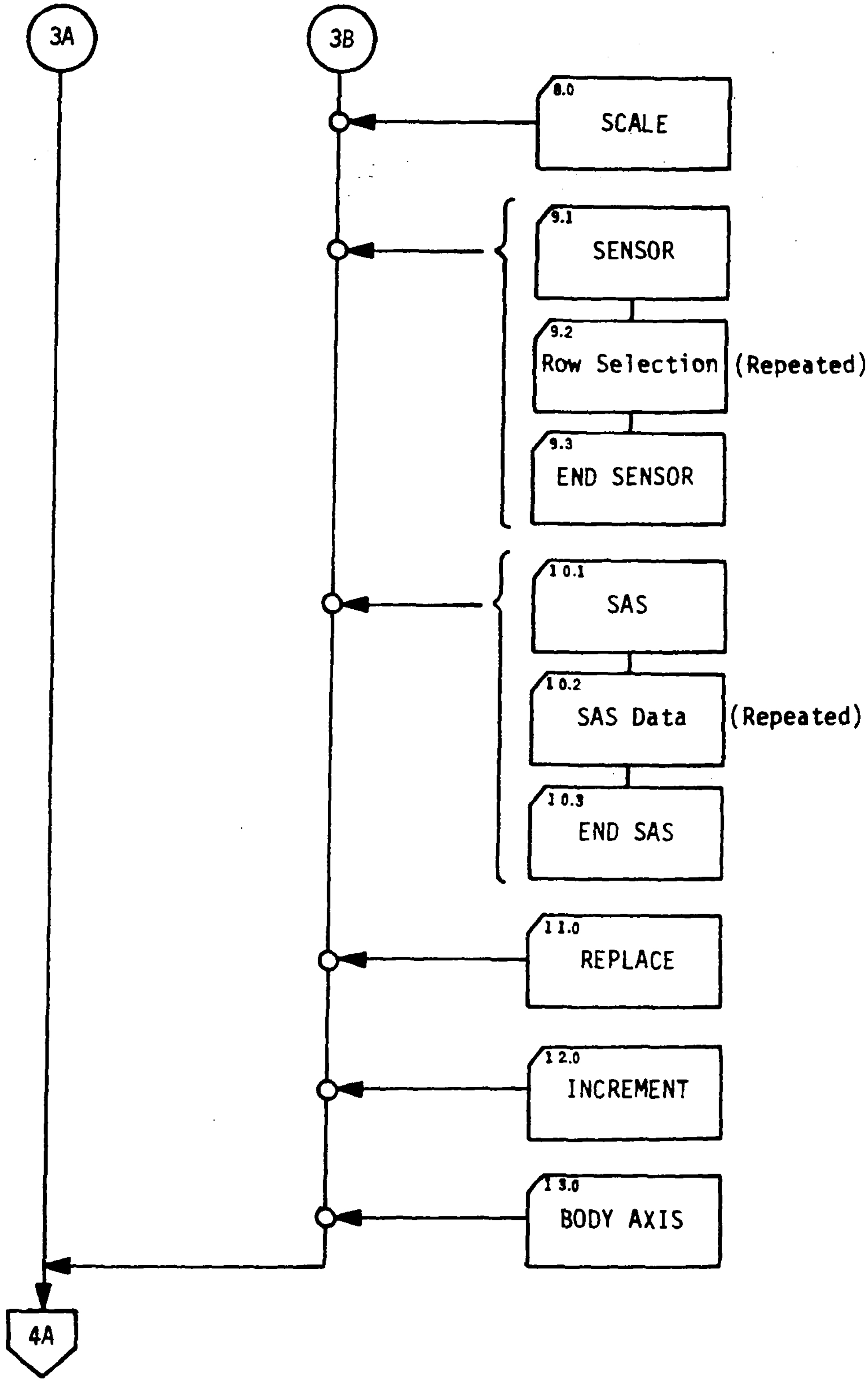


Figure 7. – (Continued)

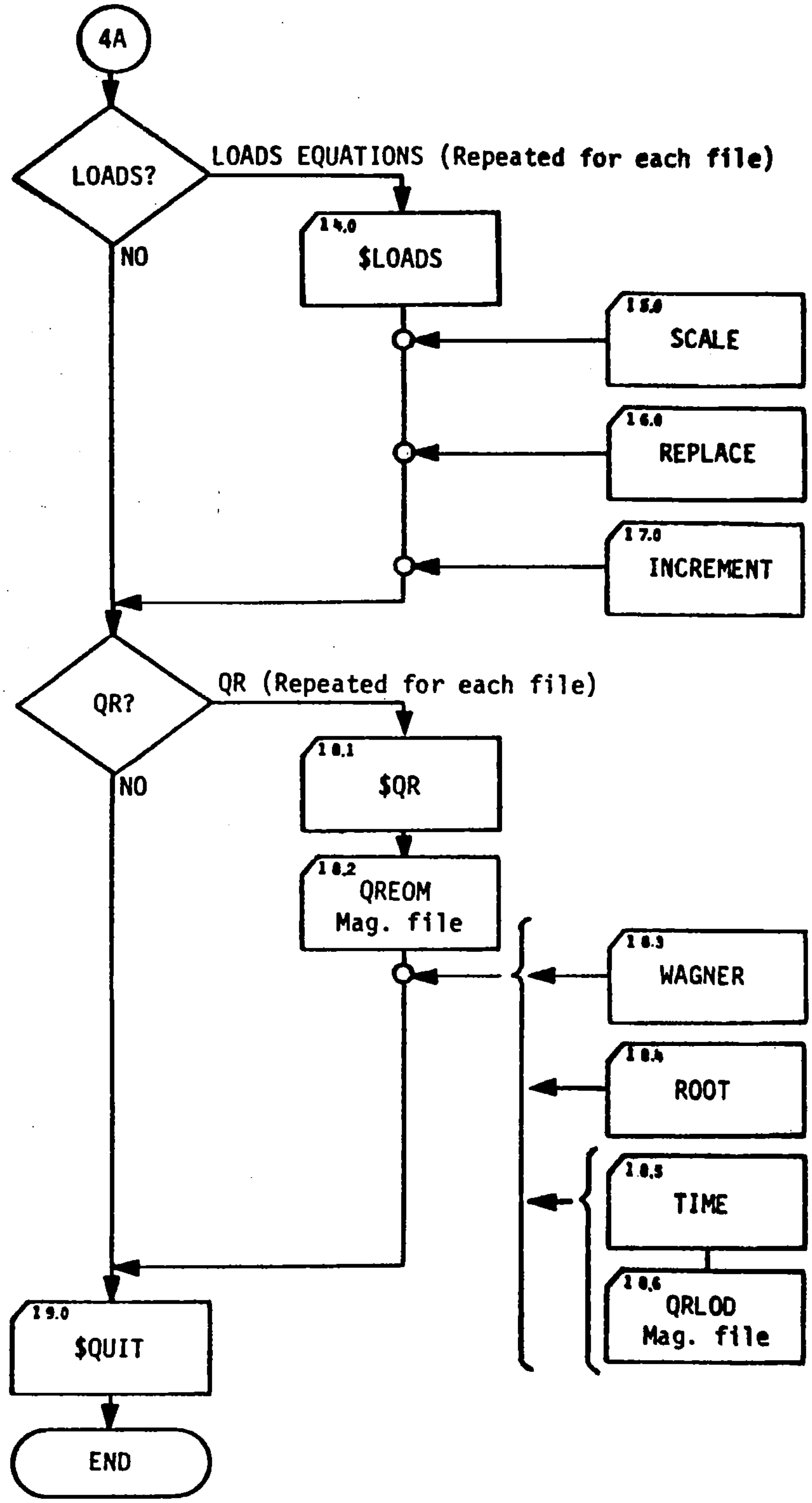


Figure 7. – (Concluded)

Format of Card Input Data

All card data is read in fixed fields, specific columns of the cards. On the pages that follow, the required card columns are defined next to each keyword or variable. The following conventions, which are used throughout the program, should be noted.

- All floating point variables are read with format E10.0.
- All integer variables are read with the format I5.
- All hollerith variables (keywords, etc.) are read with the format A10; however, when the program is trying to recognize keywords, it checks only the first four characters.

All data fields end on a card column that is a multiple of five.

6.3.1 GENERAL OPTIONS

The first card read by L219 (EQMOD) must be $EQMOD, card set 1.0. It indicates that card data for L219 (EQMOD) follows.

After Card Set 1.0 the program continues to read data cards and checks the first four characters for keywords. The keywords introduce the remaining card sets. Card sets 2.0 through 5.0 define the problem size, file names, and options to be used throughout the program execution.

Card Set 1.0 - Equation Modifier (EQMOD, L219) for Equations of Motion and Load Equations

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	$EQMod	A4,6X	This card must be the first card read by the Equation Modifier (EQMOD) program. The $EQMOD card indicates that the data following is for the Equation Modifier Program.

Card Set 2.0 - Case Labeling Information

Card 2.1 - Title Card

The title card after the $EQMOD card will be stored in core, up to four title cards, for page headings on printouts.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	TITLe	A4,6X	Keyword for job title
11-80	$TITLE_{i=1,7}$	7A10	Job title. Used to provide description of the job.

Card 2.2 – Comment Card (Optional)

Comment cards may appear anywhere in the input data stream except where data follows a keyword card.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-2	C	A2	Keyword for comment card. NOTE: A blank in column 2 must follow the C in column 1.
3-80	COMMENT	A8,7A10	Comments will appear in the printed output as they are read. It is not treated as data.

Card Set 3.0 – Problem Size

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	SIZE	A4,6X	Keyword indicating the problem size.
11-15	NDOF	I5	Total number of output degrees of freedom (no default) ($\leq$ 100)
16-20	NPAN	I5	Total number of output panels (number of gust zones) NOTE: If the equations of motion are generated external to DYLOFLEX set NPAN = 0 if no gradual penetration is being used and C_3 is real. (Default: NPAN extracted from EOM tape) ($\leq$ 50)
21-25	NFREQM	I5	Number of frequencies at which unsteady aerodynamics are defined. (Default: NFREQM extracted from EOM tape) ($\leq$ 20)

Card Set 4.0 - Output Options

Card 4.1 - Output Tapes

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	OUTPut	A4,6X	Keyword indicating output file names and file positions.
11-20	IUTEOM	A7,3X	File name where equations of motion matrices are written. (Default: IUTEOM = EQEOM)
21-25	IFLEOM	I5	File position number where equations of motion matrices are written. (Default: IFLEOM = 1)
26-30	dummy	5X	Blanks.
31-40	IUTLOD	A7,3X	File name where load equations matrices are written. (Default: IUTLOD = EQLOD)
41-45	IFLLOD	I5	File position number where load equations matrices are written. (Default: IFLLOD = 1)

Card 4.2 - Print Input Matrices (Optional)

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	PRINt	A4,6X	Keyword introducing the print option.
11-20	INPUt	A4,6X	Keyword indicating the input matrices to be printed.
21-30	MATRIX	10X	Descriptive word (optional)
31-40	OPTION	A10	Print options: Keyword — Matrices Printed ALL — All input matrices printed NONE — No input matrix printed FREQUENCY — Only input matrices of Ith frequency (ITHF) printed. Default: (OPTION = NONE)
41-45	ITHF	I5	(Required only if OPTION = FREQUENCY) Ith frequency input matrices to be printed. (Default: ITHF = 1)

Card 4.3 - Print Output Matrices (Optional)

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	PRINt	A4,6X	Keyword introducing the print option.
11-20	OUTPut	A4,6X	Keyword indicating the output matrices to be printed.
21-30	MATRIX	10X	Descriptive word (optional)
31-40	OPTION	A10	Print options: Keyword — Matrices Printed ALL — All output matrices printed NONE — No output matrices printed FREQUENCY — Only output matrices of Ith frequency (ITHF) printed. CHANGED — Only those matrices that have been changed are printed. (Default: OPTION = CHANGED)
41-45	ITHF	I5	(Required only if OPTION = FREQUENCY) Ith frequency output matrices to be printed. (Default: ITHF = 1)

Card Set 5.0 - Symmetric or Antisymmetric Analysis (Optional)

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	{ SYMMetric ANTIsymmetric }	A4,6X	Keyword SYMMETRIC indicates a symmetric analysis for body axis and derivatives. Keyword ANTISYMMETRIC indicates an anti-symmetric analysis for body axis and derivatives. (Default: SYMMETRIC)

6.3.2 INSTRUCTIONS TO MODIFY EOM MATRICES

Omit cards sets 6.0 through 13.0 if no equations of motion matrices are to be modified.

Card sets 6.0 through 13.0 contain operational instructions and data used to modify the equations of motion for use in the solution program L221 (TEV156) or any other program that is compatible with these output results.

Card Set 6.0 - Equations of Motion Data

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	$EOM	A4,6X	Keyword introducing the data for equationsof motion
11-20	INEOM	A7,3X	File name where input equationsof motion matrices reside. (Default: INEOM = EOMTAP)
21-25	INEOMF	I5	File position number where equationsof motion matrices resides. (Default: INEOMF = 1)
26-65	DYLOFLEX or $NULEOM_I$	A4,36X or 8I5	Keyword DYLOFLEX indicates that the null matrix indicator array is read from the file on which the equations of motion matrices reside. Otherwise, the null matrix indicator array is read from this card. $NULEOM_I$ = 0, matrix is null and omitted from file INEOM. $NULEOM_I \neq 0$, matrix is to be read from file INEOM. $NULEOM_1$ corresponds to M_1 $NULEOM_2$ corresponds to M_2 $NULEOM_3$ corresponds to M_3 $NULEOM_4$ corresponds to M_4 $NULEOM_5$ corresponds to M_5 $NULEOM_6$ corresponds to C_3 $NULEOM_7$ corresponds to f_ℓ $NULEOM_8$ corresponds to $\tilde{\phi}$ (No default)

Card Set 7.0 – Stability Derivative Data (Optional)

Card 7.1 – Derivative (Stability) Data

If this option is used and the gust zones (NPAN, card set 3.0) = 1 (no gust penetration), the forcing function coefficient matrix $\tilde{\phi}$ (NPAN=1) or C_3 (NPAN=0) is modified to be compatible with the response generalized forces M_4 and M_5. However, if the gust zones > 1 (gust penetration), the forcing function coefficient matrix $\tilde{\phi}$ is not modified and should be modified manually by using card sets 11.0 or 12.0 to be consistent with the response generalized forces. If the gust coefficient modification is not performed, errors may result in the responses and loads of the coordinate.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	DERIVATIVE	A4,6X	Keyword introducing derivative (stability) data.
11-20	FROM	10X	Descriptive word (preposition)
21-30	{CARD / TAPE}	A4,6X	Keyword CARD indicates derivatives are to be input on cards 7.5.1 through 7.7.2. Keyword TAPE indicates derivatives are to be input on a FLEXSTAB* SDSSTP file (card 7.4.1).
31-35	NCS	I5	Number of control surfaces. If control surface derivatives are requested (NCS > 0) read card 7.4.2 for control surface names. (Default: NCS = 0) Maximum = 20.
36-40	INDUN	I5	Indicator to request unsteady derivatives. INDUN = 0, do not read unsteady derivatives INDUN ≠ 0, read cards 7.8.1-7.8.2 for unsteady derivatives (Default: INDUN = 0)
41-50	QUEBAR	E10.0	Dynamic pressure, $\bar{q}$, (force/length^2)** (Default: QUEBAR from EOMTAP)
51-60	VT	E10.0	Velocity, true air speed, V_T, (length/sec.)** (Default: VT from EOMTAP)

***Indicates the FLEXSTAB program with the DYLOFLEX modifications is incorporated into the SD&SS program.**

****The units of force and length must be consistent and identical throughout this program and the units of the input matrices.**

Card 7.2 – Input Matrix Column Numbers of Rigid Body Freedoms

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	$\begin{Bmatrix}\text{IXCOL}\\\text{IYCOL}\end{Bmatrix}^*$	I5	Column number of the $\begin{Bmatrix}X\\Y\end{Bmatrix}$ freedoms (Default: column element not changed)
6-10	$\begin{Bmatrix}\text{IZCOL}\\\text{IPCOL}\end{Bmatrix}^*$	I5	Column number of the $\begin{Bmatrix}Z\\\phi\end{Bmatrix}$ freedoms (Default: column element not changed)
11-15	$\begin{Bmatrix}\text{ITCOL}\\\text{ISCOL}\end{Bmatrix}^*$	I5	Column number of the $\begin{Bmatrix}\theta\\\psi\end{Bmatrix}$ freedoms (Default: column element not changed)
16-70	IDCOL_I	11I5 (If more cards are needed, FORMAT for cards following is 14I5)	Column number of the δ_I control surface freedom. (I = 1, NCS see card 7.1) (Default: column element not changed)

*Throughout cards 7.2 through 7.8.1, the upper number in brackets is for symmetric analysis, and the lower number is for antisymmetric.

If any column numbers are left blank, no changes will be made to the matrix elements of those freedoms.

Card 7.3 – Constants Associated with Derivatives

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	ΔX	E10.0	X distance from the stability derivative reference point to the rigid body motion reference point, + AFT (length)** (Default: Δx = 0.0) (See fiqure 4.0)
11-20	ΔZ	E10.0	Z distance from the stability derivative reference point to rigid body motion reference point, + up (length)** (Default: Δz = 0.0) (See fiquire 4.0)
21-30	ALPHA1	E10.0	1G angle of attack, α, (degrees) (Default: ALPHA1 = 0.0)
31-40	SW	E10.0	Wing reference area, SW ($length^2$)** (Defaults: If CARD* - Fatal Error If TAPE* - Value from SDSSTP)
41-50	{CBAR / B}	E10.0	Reference chord, $\bar{c}$, (length)** Reference span, b, (length)** (Defaults: If CARD* - Fatal Error If TAPE* - Value from SDSSTP)
51-60	CL1R	E10.0	"RIGID" steady state derivative, C_{L1R} (Defaults: If CARD* - CL1R = 0 and warning message printed. If TAPE* - Value from SDSSTP)
61-70	CL1E	E10.0	"ELASTIC INCREMENT" steady state derivative, C_{L1E} (same defaults at C_{L1R}

*Note: Keyword CARD or TAPE is defined on Card 7.1, cols. 21-30.

**See note on Card 7.1

Card 7.4.1 – Files Containing FLEXSTAB Aerodynamic Data

Read this card only if columns 21 through 30 are TAPE on card 7.1.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	SDSStp	A10	Keyword introducing replacement name for the FLEXSTAB aerodynamic data file.
11-20	SDINDX	A10	Name of the file (tape or disk) containing the index table to the FLEXSTAB aerodynamic data. The name must begin in column eleven, begin with a letter, and have less than seven characters. Default: SDINDX = "SDINDX"
21-30	SDDATA	A10	Name of the file (tape or disk) containing the FLEXSTAB aerodynamic data. NOTE: SDINDX ≠ SDDATA. The name must begin in column 21, begin with a letter and have less then seven characters. Default: SDDATA = "SDDATA"
31-35	ISCAS	I5	FLEXSTAB aerodynamic data case number from which L217 (EOM) must extract derivative. Default: ISCAS = 1
36-70			Available for comments

Card 7.4.2 – Derivatives from File per Control Surface

Read this card if columns 21 through 30 are TAPE, and NCS > 0, on card 7.1. Then go to card set 7.8 for unsteady derivatives.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-70	NAMESCI	7A10 (Repeat this card if more names are needed)	User defined active control surface names associated with active control surface derivatives. (I = 1, NCS see card 7.1) Names are first defined in FLEXSTAB.

Note: Read cards 7.5.1 through 7.8.2 only if keyword CARD (card 7.1) is selected.

Card 7.5.1 – RIGID Derivatives from Cards

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	{CLU / CYB}	E10.0	Steady state derivative, $\begin{cases} C_{L\hat{u}_R} \\ C_{y\beta} \end{cases}$
11-20	{CDU / CLBREF}	E10.0	Steady state derivative, $\begin{cases} C_{D\hat{u}_R} \\ C_{\ell\beta_{REF}} \end{cases}$
21-30	{CMUREF / CNBREF}	E10.0	Steady state derivative, $\begin{cases} C_{m\hat{u}_{REF}} \\ C_{n\beta_{REF}} \end{cases}$
31-40	{CLA / CYP}	E10.0	Steady state derivative, $\begin{cases} C_{L\alpha_R} \\ C_{y\hat{p}} \end{cases}$
41-50	{CDA / CLPREF}	E10.0	Steady state derivative, $\begin{cases} C_{D\alpha_R} \\ C_{\ell\hat{p}_{REF}} \end{cases}$
51-60	{CMAREF / CNPREF}	E10.0	Steady state derivative, $\begin{cases} C_{m\alpha_{REF}} \\ C_{n\hat{p}_{REF}} \end{cases}$

Card 7.5.2 – ELASTIC INCREMENT Derivatives Read from Cards

Read this card for SYMMETRIC analysis only (card set 5.0).

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	CLUE	E10.0	Steady state derivative, $C_{L\hat{u}_E}$
11-20	CDUE	E10.0	Steady state derivative, $C_{D\hat{u}_E}$
21-30	dummy		
31-40	CLAE	E10.0	Steady state derivative, $C_{L\alpha_E}$
41-50	CLAE	E10.0	Steady state derivative, $C_{D\alpha_E}$

Card 7.6.1 - RIGID Derivatives Read from Cards

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	$\begin{Bmatrix}\text{CLQ}\\ \text{CYR}\end{Bmatrix}$	E10.0	Steady state derivative, $\begin{cases} C_{L\hat{q}_R} \\ C_{Y\hat{r}} \end{cases}$
11-20	$\begin{Bmatrix}\text{CDQ}\\ \text{CLRREF}\end{Bmatrix}$	E10.0	Steady state derivative, $\begin{cases} C_{D\hat{q}_R} \\ C_{\ell\hat{r}_{REF}} \end{cases}$
21-30	$\begin{Bmatrix}\text{CMQREF}\\ \text{CNRREF}\end{Bmatrix}$	E10.0	Steady state derivative, $\begin{cases} C_{m\hat{q}_{REF}} \\ C_{n\hat{r}_{REF}} \end{cases}$

Card 7.6.2 - ELASTIC INCREMENT Derivatives Read from Cards

Read this card for SYMMETRIC analysis only (card set 5.0).

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	CLQE	E10.0	Steady state derivative, $C_{L\hat{q}_E}$
11-20	CDQE	E10.0	Steady state derivative, $C_{D\hat{q}_E}$

Card 7.7.1 - RIGID Derivatives Read from Cards per Control Surface

Repeat card 7.7.1 and 7.7.2 in pairs NCS times (card 7.1.). If NCS = 0, omit this card.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	$\begin{Bmatrix}\text{CLD}_I\\ \text{CYD}_I\end{Bmatrix}$	E10.0	Steady state derivative, $\begin{cases} C_{L\delta_{R_I}} \\ C_{Y\delta_I} \end{cases}$
11-20	$\begin{Bmatrix}\text{CDD}_I\\ \text{CLDREF}_I\end{Bmatrix}$	E10.0	Steady state derivative, $\begin{cases} C_{D\delta_{R_I}} \\ C_{\ell\delta_{REF_I}} \end{cases}$
21-30	$\begin{Bmatrix}\text{CMDREF}_I\\ \text{CNDREF}_I\end{Bmatrix}$	E10.0	Steady state derivative, $\begin{cases} C_{m\delta_{REF_I}} \\ C_{n\delta_{REF_I}} \end{cases}$

Card 7.7.2 – ELASTIC INCREMENT Derivatives Read from Cards per Con

Read this card for SYMMETRIC analysis only (card set 5.0).

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	$CLDE_I$	E10.0	Steady state derivative, $C_{L\delta_{R_I}}$
11-20	$CDDE_I$	E10.0	Steady state derivative, $C_{D\delta_{R_I}}$
			(I = 1, NCS)

Card 7.8.1 – RIGID Unsteady Derivatives Read from Card

Read this card if INDUN ≠ 0 (card 7.1).

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	{CLADOT / CYBDOT}	E10.0	Unsteady derivative, $\left\{\begin{matrix} C_{L\hat{\alpha}_R} \\ C_{y\hat{\beta}} \end{matrix}\right.$
11-20	{CDADOT / CLBDRF}	E10.0	Unsteady derivative, $\left\{\begin{matrix} C_{D\hat{\alpha}_R} \\ C_{\ell\hat{\beta}_{REF}} \end{matrix}\right.$
21-30	{CMADRF / CNBDRF}	E10.0	Unsteady derivative, $\left\{\begin{matrix} C_{m\hat{\alpha}_{REF}} \\ C_{n\hat{\beta}_{REF}} \end{matrix}\right.$

Card 7.8.2 – ELASTIC INCREMENT Unsteady Derivatives Read from Car

NOTES: 1. Read this card for SYMMETRIC analysis only (card set 5.0)

2. Read this card if INDUN = 0 (card 7.1).

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	CLADTE	E10.0	Unsteady derivative, $C_{L\hat{\alpha}_E}$
11-20	CDADTE	E10.0	Unsteady derivative, $C_{D\hat{\alpha}_E}$

Card Set 8.0 - Scale EOM Matrix Elements (Optional)

This card is repeated for each EOM matrix to be scaled.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	SCALe	A10	Keyword introducing matrix elements to be scaled.
11-20	MATNAM	A10	Equations of motion matrix name to be scaled. Matrix name must be one of the following keywords: M1, M2, M3, FREQ, M4, M5, C3, FL, PHI
21-25	IFREQ	I5	Matrix of Ith frequency (Default: IFREQ = 1)
26-30	dummy	5X	Blanks
31-40	SCLMAT	E10.0	Scalar by which each element of this matrix is multiplied.

Card Set 9.0 - Sensor Data (Optional)

Card 9.1 - Introduce Sensor Data

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	SENSor	A4,6X	Keyword introducing sensor data
11-20		A7,3X	Name of file on which the input sensor matrices reside. (Default: INSEN = LODTP2)
21-25	INSENF	I5	File position number in which sensor matrices reside. (Default: INSENF = 1)
26-30	NLDSEN	I5	Number of loads (row size) of the sensor matrices being read from INSEN file. (No default)
31-45	DYLOFLEX or $NULSEN_I$	A4,11X or 3I5	Keyword DYLOFLEX indicates that the null matrix indicator array is read from the file on which the sensor matrices reside. Otherwise, the null matrix indicator array is read from this card: $NULSEN_I$ = 0, matrix is null and omitted from the file. $NULSEN_I \neq 0$, matrix is read from file. $NULSEN_1$ corresponds to M_1 $NULSEN_2$ corresponds to M_2 $NULSEN_3$ corresponds to M_3 (no default)

Card 9.2 – Row Selection of Sensor Data

Repeat this card for each matrix from which sensor rows are selected.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	M1BAr M2BAr M3BAr	A4,6X	Matrix name of sensor data. Matrix name must be one of the following keywords: M1BAR, M2BAR, M3BAR (No default)
11-15	INROW	I5	The row number of the input matrix where sensor data will be selected. (No default)
16-20	IUTROW	I5	The row number of the output (augmented) matrix where the selected sensor data will be placed. (No default)
21-25	INROW	I5	
26-30	IUTROW	I5	Each pair of input and output row numbers is repeated
31-35	INROW	I5	for each row selection.
36-40	IUTROW	I5	If more than six rows selected, repeat this card
41-45	INROW	I5	with the same matrix name.
46-50	IUTROW	I5	NOTE: a "-1." is also placed in the "IUTROW" row -
51-55	INROW	I5	column diagonal element of the M1 equation of
56-60	IUTROW	I5	motion matrix.
61-65	INROW	I5	
66-70	IUTROW	I5	

Card 9.3 – End Sensor Data

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	END sensor	A4,6X	Keyword indicating the end of the sensor data.

Card Set 10.0 - SAS Data (Optional)

Card 10.1 - Introduce SAS Data

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	SAS	A4,6X	Keyword introducing SAS data.

Card 10.2 - SAS Data

This card is repeated as many times as necessary in order to define the elements of the stability augmentation system (SAS) equations which are to be placed in the equations of motion.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-5	ISAS	I5	Ith row of augmented equation to the equations of motion (Default: Pickup previous row number; exception, the first ISAS has no default.)
6-10	JSAS	I5	Jth column of augmented equation to the equations of motion (No default)
11-20	M1IJ	E 10.0	Value of the element of the SAS equation to be placed in the Ith, Jth location of the M_1 matrix.
21-30	M2IJ	E 10.0	Value of the element of the SAS equation to be placed in the Ith, Jth location of the M_2 matrix.
31-40	M3IJ	E 10.0	Value of the element of the SAS equation to be placed in the Ith, Jth location of the M_3 matrix.

Card 10.3 - End SAS Data

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	END sas	A4,6X	Keyword indicating all SAS data has been defined.

Card Set 11.0 – Replace EOM Matrix Elements (Optional)

This card may be repeated.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	REPLace	A6,6X	Keyword introducing matrix elements to be replaced.
11-20	MATNAM	A10	Equationsof motion matrix name to be replaced. Matrix name must be one of the following keywords: M1, M2, M3, FREQ, M4, M5, C3, FL, PHI
21-25	IFREQ	I5	Matrix of Ith frequency (Default: IFREQ = 1)
26-30	dummy	5X	Blanks
31-35	IROW	I5	Ith row of matrix
36-40	JCOL	I5	Jth column of matrix
41-50	AIJ	E10.0	Value replacing Ith, Jth element of matrix

Card Set 12.0 – Increment EOM Matrix Elements (Optional)

This card may be repeated.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	INCRement	A4,6X	Keyword introducing matrix elements to be incremented
11-20	MATNAM	A10	Equation of motion matrix name to be incremented. Matrix Name must be one of the following keywords: M1, M2, M3, FREQ, M4, M5, C3, FL, PHI
21-25	IFREQ	I5	Matrix of Ith frequency (Default: IFREQ = 1)
26-30	dummy	5X	Blanks
31-35	IROW	I5	Ith row of matrix
36-40	JCOL	I5	Jth column of matrix
41-50	AIJ	E10.0	Value incrementing Ith, Jth element of matrix

If the user wishes to transform the equations of motion and load equations from inertial axes to body-fixed axes, this card set is used. This card set is not repeated.

Card Set 13.0 - Body Axis Transformation (Optional)

For this card set, the upper number in brackets is for the symmetric analysis, and the lower number is for the antisymmetric analysis.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	BODYaxis	A4,6X	Keyword introducing body axis transformation data
11-15	$\begin{Bmatrix} \text{ICOLX} \\ \text{ICOLY} \end{Bmatrix}$	I5	$\begin{Bmatrix} X_{COL} \\ Y_{COL} \end{Bmatrix}$ column of the original axis (Default: column elements not used)
16-20	$\begin{Bmatrix} \text{ICOLZ} \\ \text{ICOLP} \end{Bmatrix}$	I5	$\begin{Bmatrix} Z_{COL} \\ \phi_{COL} \end{Bmatrix}$ column of the original axis (No default)
21-25	$\begin{Bmatrix} \text{ICOLT} \\ \text{ICOLS} \end{Bmatrix}$	I5	$\begin{Bmatrix} \theta_{COL} \\ \psi_{COL} \end{Bmatrix}$ column of the original axis (No default)
26-30	dummy	5X	Blanks
31-40	ALPHA1	E10.0	α_1 1G angle of attack (degrees) (Default: ALPHA1 = 0.0)
41-50	BODYVT	E10.0	Velocity, true air speed, V_T, (length/sec.)* (Default: Body V_T from EOMTAP)

*See note on card 7.1.

6.3.3 INSTRUCTIONS TO MODIFY LOADS MATRICES

Omit card sets 14.0 through 17.0 if no load equation matrices are to be modified.

Card sets 14.0 through 17.0 contain operational instructions and data used to modify the load equations for use in the solution program L221 (TEV156) or any other program that is compatible with these output results.

Card Set 14.0 - LOADS Equations Data

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	$LOAds	A4,6X	Keyword introducing the data for load equations
11-20	INLOD	A7,3X	File name where input load equationsmatrices reside. (Default: INLOD = LODTAP)
21-25	INLODF	I5	File position number where load equationsmatrices reside (Default: INLODF = 1)
26-30	NLDOU	I5	Total number of output loads (No default)
31-65	{DYLOFLX / $NULLOD_I$}	{A4,3X / 7I5}	Keyword DYLOFLX indicates that the null matrix indicator array is read from the file on which the load equations matrices reside. Otherwise, the null matrix indicator array is read from the same card columns. $NULLOD_I$ = 0, matrix is null and omitted from the file. $NULLOD_I \neq 0$, matrix is read from file. $NULLOD_1$ corresponds to $\bar{M}_1$ $NULLOD_2$ corresponds to $\bar{M}_2$ $NULLOD_3$ corresponds to $\bar{M}_3$ $NULLOD_4$ corresponds to $\bar{M}_4$ $NULLOD_5$ corresponds to $\bar{M}_5$ $NULLOD_6$ corresponds to $\bar{C}_3$ $NULLOD_7$ corresponds to $\bar{\Phi}$

Note: Card set 14.0 is required if SAS equations are added to the equations of motion. The use of card set 14.0 will increase the column size of the load coefficient matrices.

Card Set 15.0 - SCALE LOADS Matrix Elements (Optional)

Repeat this card for each LOADS matrix to be scaled.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	SCALe	A10	Keyword introducing matrix elements to be scaled.
11-20	MATNAM	A10	Load equation matrix name to be scaled, matrix name must be one of the following keywords: M1BAR, M2BAR, M3BAR, M4BAR, M5BAR, C3BAR, PHIBAR
20-25	IFREQ	I5	Matrix of Ith frequency (Default: IFREQ = 1)
26-30	dummy	5X	Blanks
31-40	SCLMAT	E10.0	Scalar, multiply each element of this matrix by this value.

Card Set 16.0 - Replace LOADS Matrix Elements (Optional)

This card may be repeated.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	REPLace	A4,6X	Keyword introducing matrix elements to be replaced.
11-20	MATNAM	A10	Loads Equation matrix name to be replaced. Matrix name must be one of the following keywords: M1BAR, M2BAR, M3BAR, M4BAR, M5BAR, C3BAR, PHIBAR
21-25	IFREQ	I5	Matrix of Ith frequency (Default: IFREQ = 1)
26-30	dummy	5X	Blanks
31-35	IROW	I5	Ith row of matrix
36-40	JCOL	I5	Jth column of matrix
41-50	AIJ	E10.0	Value replacing Ith, Jth element of matrix

Card Set 17.0 - Increment LOADS Matrix Elements (Optional)

This card may be repeated.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	INCRement	A4,6X	Keyword introducing matrix elements to be incremented.
11-20	MATNAM	A10	Load equation matrix name to be incremented matrix name must be one of the following keywords: M1BAR, M2BAR, M3BAR, M4BAR, M5BAR, C3BAR, PHIBAR
21-25	IFREQ	I5	Matrix of Ith frequency (Default: IFREQ = 1)
26-30	dummy	5X	Blanks
31-35	IROW	I5	Ith row of matrix
36-40	JCOL	I5	Jth column of matrix
41-50	AIJ	E10.0	Value incrementing Ith, Jth element of matrix

6.3.4 INSTRUCTIONS FOR PREPARATION OF QR MATRICES

Omit card set 18.0 if no QR matrices are to be generated.

Card set 18.0 contains operational instructions and data used to modify the equations of motion and load equations for use in the solution program QR or any other program that is compatible with these output results.

Card Set 18.0 - QR Data Preparation (Optional)

Repeat cards 18.1 and 18.2 for each of either cards 18.3 or 18.4 or 18.5, and 18.6

1. QR data may be executed with the modified or unmodified EOM and LOADS matrices as specified on cards 18.1 and 18.6 respectively.
2. QR data may be executed without reading card sets 4.0 through 17.0

Card 18.1 - Request QR Data Preparation

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	$QR	A4,6X	Keyword introducing the data for QR output
11-20	IUTQR	A7,3X	File name where QR matrices are to be written. (Default: IUTQR = QRTAP)
21-25	IFLQR	I5	File position number where QR matrices are to be written. (Default: IFLQR = 1)

Card 18.2 – Input Equations of Motion Matrices for QR

COLS.	KEYWORD/ VARIBLE	FORMAT	DESCRIPTION
1-10	QREOm	A4,6X	Keyword introducing the source of the equationsof motion matrices.
11-20	INEOM	A7,3X	File name where input equationsof motion matrices reside. (Default: INEOM = EQEOM)
21-25	INEOMF	I5	File position number where equationsof motion matrices resides (Default: INEOMF = 1)
26-65	$\begin{Bmatrix}\text{DYLOFLX}\\ \text{NULEOM}_I\end{Bmatrix}$	$\begin{Bmatrix}\text{A4,36X}\\ \text{8I5}\end{Bmatrix}$	Keyword DYLOFLX indicate null matrix indicator array is read from file where equationsof motion matrices reside. Otherwise, null matrix indication array is read from cards. $NULEOM_I$ = 0, matrix is null and omitted from the file. $NULEOM_I$ ≠ 0, matrix is read from file $NULEOM_1$ corresponds to M_1 $NULEOM_2$ corresponds to M_2 $NULEOM_3$ corresponds to M_3 $NULEOM_4$ corresponds to M_4 $NULEOM_5$ corresponds to M_5 $NULEOM_6$ corresponds to C_3 $NULEOM_7$ corresponds to f_ℓ $NULEOM_8$ corresponds to $\tilde{\Phi}$ (No default)

Card 18.3 – QR Wagner Option

This card is used if roots with Wagner indical lift growth functions are to be calculated by QR.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	WAGNer	A4,6X	Keyword indicating QR matrices are to be formulated using the equations of motion with Wagner functions.
11-20	AQR	E10.0	Wagner function, a_1 (see equation I5)
21-30	BQR	E10.0	Wagner function, b_1 (see equation I5)
31-40	ALQR	E10.0	Wagner function, α_1 (see equation I5)
41-50	BEQR	E10.0	Wagner function, β_1 (see equation I5)

Card 18.4 – QR Root Option

This card is used if roots without Wagner indical lift growth function are to be calculated by QR.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	ROOT	A4,6X	Keyword indicating QR matrices are to be formulated using the equations of motion without Wagner functions.

Card 18.5 – QR TIME Option

This card is used if time histories are to be calculated by QR.

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	TIME	A4,6X	Keyword indicating QR matrices are to be formulated using the equations of motion and load equation matrices for a time history solution.

Card 18.6 – Input Load Equations Matrices for QR

This card is required if keyword TIME (card 18.5) is used.

COLS.	KEYWORD/ VARABLE	FORMAT	DESCRIPTION
1-10	QRLOad	A4,6X	Keyword introducing the source of the load equation matrices.
11-20	INLOD	A7,3X	File name where input load equation matrices reside (Default: INLOD = EQLOD)
21-25	INLODF	I5	File position number where load equationsmatrices reside (Default: INLODF = 1)
26-30	NLDQR	I5	Number of loads (Default: If DYLOFLX file, NLDQR is extracted from the first record on the file INLOD).
31-65	DYLOFLX $NULLOD_I$	A4,3X 7I5	Keyword DYLOFLX indicate null matrix indicator array is read from file where load equationsmatrices reside. Otherwise, null matrix indicator array is read from this card. $NULLOD_I$ = 0, matrix is null and omitted from the file INLOD. $NULLOD_I$ ≠ 0, matrix is to be read from file INLOD $NULLOD_1$ corresponds to M_1 $NULLOD_2$ corresponds to M_2 $NULLOD_3$ corresponds to M_3 $NULLOD_4$ corresponds to M_4 $NULLOD_5$ corresponds to M_5 $NULLOD_6$ corresponds to C_3 $NULLOD_7$ corresponds to Φ

Card Set 19.0 – Terminator

COLS.	KEYWORD/ VARIABLE	FORMAT	DESCRIPTION
1-10	$QUIt	A4,6X	Keyword indicating the last data of the EQMOD module has been read. The $QUIT card may include comments following the fourth column.

6.3.5 SUMMARY OF CARD INPUT DATA

Requirements or Function	Key Words and/or Variables	Card Format	Reference Card Set (CS)
	$EOMod	A4,6X	1.0
	TITLe Title Card	A4,6X,7A10	2.1
	C Comment Card	A2,A8,7A10	2.2
Problem Size	SIZE NDOF NPAN NFREQM	A4,6X,3I5	3.0
Output Options	OUTPut IUTEOM IFLEOM dummy IUTLOD IFLLOD	A4,6X,A7,3X, I5,5X,A7,3X,I5	4.1
Print Input Options	PRINt INPUt MATRIX OPTION ITHF	2(A4,6X), 10X,10A,I5	4.2
Print Output Options	PRINt OUTPut MATRIX OPTION ITHF	2(A4,6X), 10X,10A,I5	4.3
Indicator for Body Axis and Derivatives	{SYMMetric / ANTIsymmetric}	A4,6X	5.0

Equations of Motion Data

Requirements or Function	Key Words and/or Variables	Card Format	Reference Card Set (CS)
	$EOM INEOM INEOMF {DYLOFLX / $NULEOM_I$}	A4,6X,A7,3X I5, {A4,36X / 8I5}	6.0
Derivative Data	DERIvative FROM {CARD / TAPE} NCS INDUN QUEBAR VT	A4,6X,10X,A4,6X, 2I5,2E10.0	7.1
Column Numbers of Rigid Body Freedoms	{IXCOL / IYCOL} {IZCOL / IPCOL} {ITCOL / ISCOL} $IDCOL_I$	14I5	7.2
Derivative Constants	Δx Δz ALPHA1 SW {CBAR / B} CL1R CL1E	7E10.0	7.3

Requirements or Function	Key Words and/or Summary	Card Format	Reference Card Set (CS)
FLEXSTAB Aero Data	SDSStp SDINDX SDDATA ISCAS	3A10,I5	7.4.1
	NAMECSI	7A10	7.4.2
Rigid Derivatives	{CLU CYB} {CDU CLBREF} {CMUREF CNBREF} {CLA CYP} {CDA CLPREF} {CMAREF CNPREF}	6E10.0	7.5.1.
Elastic Increment if SYMM on CS 5.0	CLUE CDUE dummy CLAE CDAE	2E10.,10X,2E10.	7.5.2
Rigid Derivatives	{CLQ CYR} {CDQ CLRREF} {CMQREF CNRREF}	3E10.0	7.6.1
Elastic Increment if SYMM on CS 5.0	CLQE CDQE	2E10.0	7.6.2
Control Surface Rigid Derivatives	{CLD_I CYD_I} {CDD_I $CLDREF_I$} {$CMDREF_I$ $CNDREF_I$}	3E10.0	7.7.1
Elastic Increment if SYMM on CS 5.0	$CLDE_I$ $CDDE_I$	2E10.0	7.7.2
Rigid Unsteady Derivatives	{CLADOT CYBDOT} {CDADOT CLBDRF} {CMADRF CNBDRF}	3E10.0	7.8.1
Elastic Increment if SYMM on CS 5.0	CLADTE CDADTE	2E10.0	7.8.2

Requirements or Function	Key Words and/or Summary	Card Format	Reference Card Set (CS)
Scale Matrix Elements	SCALe MATNAM IFREQ dummy SCLMAT	2A10, I5,5X, E10.0	8.0
Sensor Data	SENSor INSEN INSENF NLDSEN {DYLOFLX / $NULSEN_I$}	A4,6X,A7, {A4,11X / 3X,2I5 3I5}	9.1
	{M1BAr / M2BAr / M3BAr} INROW IUTROW INROW IUTROW - - - - - - - - - - - -	A4,6X,12I5	9.2
	END sensor	A4,6X	9.3
SAS Data	SAS	A4,6X	10.1
	ISAS JSAS M1IJ M2IJ M3IJ	2I5,3E10.0	10.2
	END sas	A4,6X	10.3
Replace Matrix Elements	REPLace MATNAM IFREQ dummy IROW JCOL AIJ	A4,6X,A10,I5, 5X,2I5,E10.0	11.0
Increment Matrix Elements	INCRement MATNAM IFREQ dummy IROW JCOL AIJ	A4,6X,A10,I5, 5X,2I5,E10.0	12.0
Body Axis Transformation	BODYaxis {ICOLX / ICOLY} {ICOLZ / ICOLP} {ICOLT / ICOLS} dummy ALPHA1 BODYVT	A4,6X,3I5,5X, 2E10.0	13.0

Requirements or Function	Key Words and/or Data	Card Format	Reference Card Set (CS)
Loads Equations Data			
	$LOAds INLOD INLODF NLDOU {DYLOFLX / NULLOD$_I$}	A4,6X,A7,3X, 2I5,{A4,3X / 7I5}	14.0
Matrix Scalar	SCALe MATNAM IFREQ dummy SCLMAT	2A10,I5,5X E10.0	15.0
Replace Matrix Elements	REPLace MATNAM IFREQ dummy IROW JCOL AIJ	A4,6X,A10,I5,5X, 2I5,E10.0	16.0
Increment Matrix Elements	INCRement MATNAM IFREQ dummy IROW JCOL AIJ	A4,6X,A10,I5,5X, 2I5,E10.0	17.0
QR Data			
	$QR IUTQR IFLQR	A4,6X,A7,3X,I5	18.1
QR-EOM Input Matrices	QREOm INEOM INEOMF {DYLOFLX / NULEOM$_I$}	A4,6X,A7,3X,I5, {A4,36X / 8I5}	18.2
Wagner Option	WAGNer AQR BQR ALQR BEQR	A4,6X,4E10.0	18.3
Root Option	ROOT	A4,6X	18.4
Time Option	TIME	A4,6X	18.5
QR-Loads Equations Input Matrices	QRLOad INLOD INLODF NLDQR {DYLOFLX / NULLOD$_I$}	A4,6X,A7,3X,2I5, {A4,3X / 7I5}	18.6
	$QUIt	A4,6X	19.0

6.4 MAGNETIC FILES INPUT DATA

The input matrices to the L219 (EQMOD) program will normally be obtained from magnetic files (tape or disk) prepared by the programs; Equations of Motion, L217 (EOM), and Load Equations, L218 (LOADS). A magnetic file prepared by FLEXSTAB with the DYLOFLEX modifications will also be needed when using the stability derivatives generated in FLEXSTAB. However, because the EQMOD output magnetic files are in the same format as the input files, it is possible for EQMOD to use as input the magnetic files generated by a previous execution of EQMOD. In addition, any user generated magnetic files(s) may be used as input into EQMOD if they have the required format.

The format for the equations of motion input magnetic file is shown in figure 8. The format for the load equations input magnetic files is shown in figure 9 for the load equations, and in figure 10 for the sensor equations (which are themselves a specific type of load equation).

All EOM and LOADS matrices are in the READTP/WRTETP format.[1]

The stability derivatives generated in FLEXSTAB are written on the SDSSTP magnetic file. Before executing L219 (EQMOD), this data must be copied by EQMOD onto two magnetic files, SDINDX and SDDATA. SDINDX contains the FLEXSTAB index matrix and SDDATA the FLEXSTAB stability derivative data shown in figure 11.

6.5 OUTPUT DATA

6.5.1 PRINTED

All card input data will be printed as read and interpreted. Optionally the following data may be printed:

1. Matrices read from files

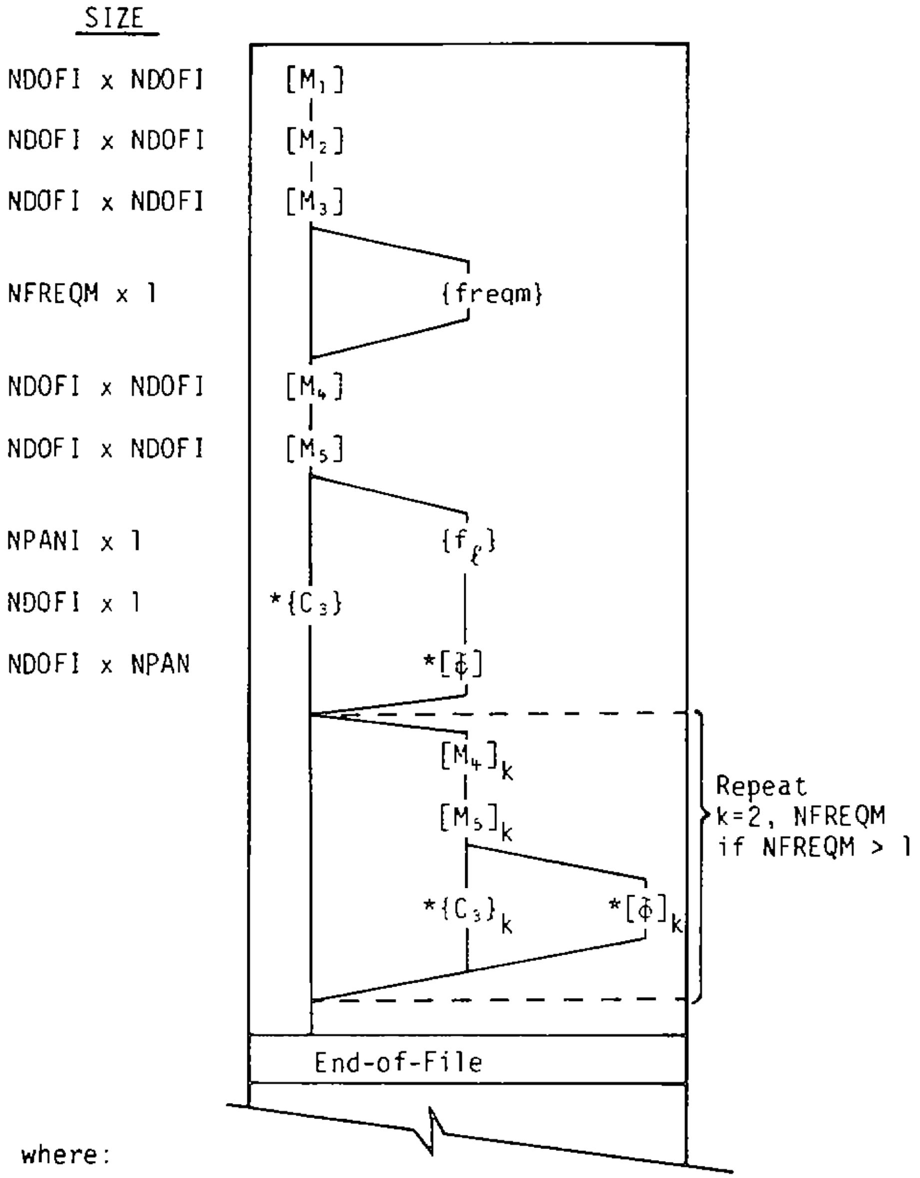
2. Matrices written on output files.

6.5.2 MAGNETIC FILES

EQMOD will write as many as three magnetic files containing modified equations of motion, modified load equations, and a file for the Linear System Analysis program, QR, which may contain the characteristic equations for the modified equations of motion and/or the modified equations of motion and load equations.

---

[1] R. E. Clemmons: *Programming Specifications for Modules of the Dynamic Loads Analysis System to Interface with FLEXSTAB*. NASA contract NAS1-13918, BCS-G0701 (internal document), September 1975.



where:

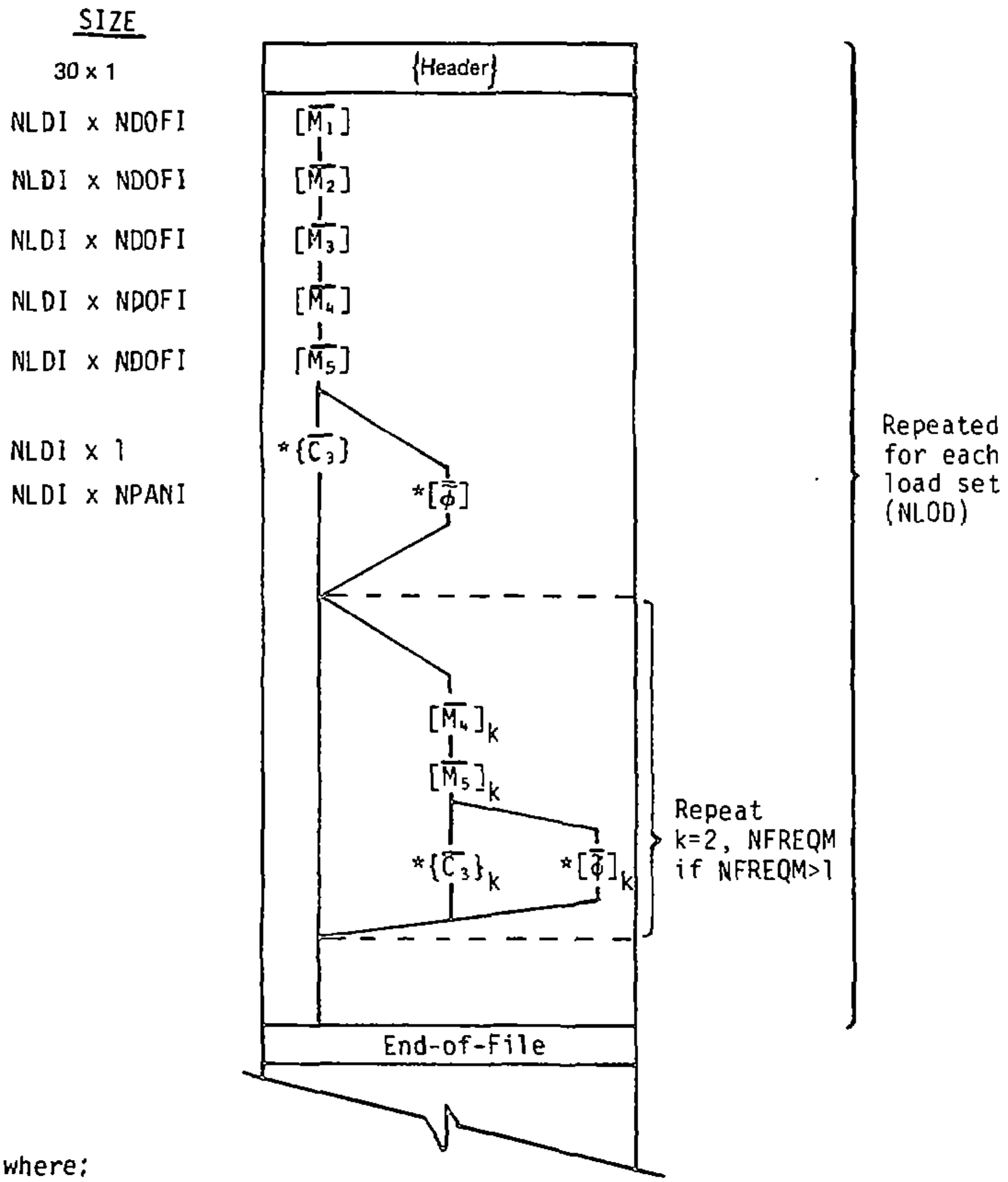
NDOFI - Number of degrees of freedom input

NFREQM - Number of frequencies

NPANI - Number of panels input

* Note: If {freqm} exists then C_3 and Φ will be complex

Figure 8. – Equations of Motion Input File (EOMTAP)

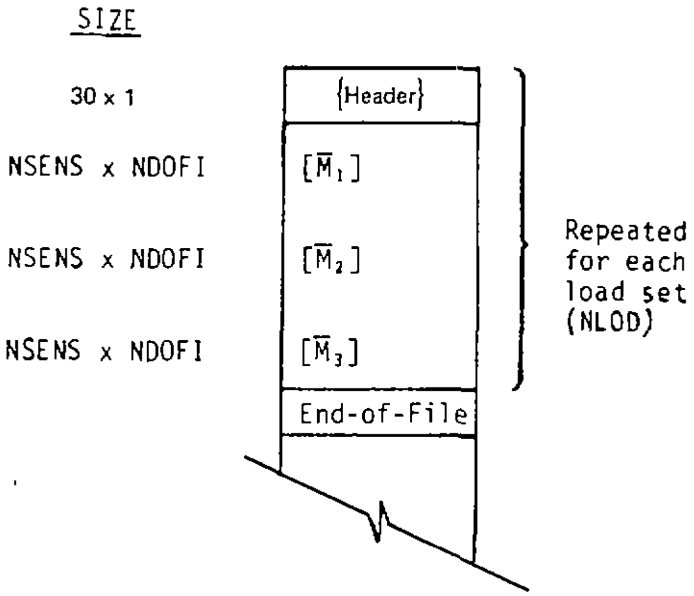


where;

NLDI - Number of loads input
NDOFI - Number of degrees of freedom input
NPANI - Number of panels input
NFREQM - Number of frequencies
NLOD - Number of load sets

* Note: If NFREQM > 1 then C_3 and $\bar{\Phi}$ will be complex

Figure 9. — Load Equations Input File (LODTAP)



where:

NSENS - Number of sensor loads

NDOFI - Number of degrees of freedom input

NLOD - Number of load sets

Figure 10. – Sensor Equations Input File (LODTP2) from the AVD Loads Path

FLEXSTAB name	Engineering symbol
(CONTRL)–A	$\begin{bmatrix} C_{Y_{\delta_{a_{\text{rigid}}}}} & C_{Y_{\delta_{a_{\text{elastic incr}}}}} & C_{Y_{\delta_{r_{\text{rigid}}}}} & C_{Y_{\delta_{r_{\text{elastic incr}}}}} & C_{Y_{\delta_{c_{i_{\text{rigid}}}}}} & C_{Y_{\delta_{c_{i_{\text{elastic incr}}}}}} \\ C_{\ell_{\delta_{a_{\text{rigid}}}}} & C_{\ell_{\delta_{a_{\text{elastic incr}}}}} & C_{\ell_{\delta_{r_{\text{rigid}}}}} & C_{\ell_{\delta_{r_{\text{elastic incr}}}}} & C_{\ell_{\delta_{c_{i_{\text{rigid}}}}}} & C_{\ell_{\delta_{c_{i_{\text{elastic incr}}}}}} \\ C_{n_{\delta_{a_{\text{rigid}}}}} & C_{n_{\delta_{a_{\text{elastic incr}}}}} & C_{n_{\delta_{r_{\text{rigid}}}}} & C_{n_{\delta_{r_{\text{elastic incr}}}}} & C_{n_{\delta_{c_{i_{\text{rigid}}}}}} & C_{n_{\delta_{c_{i_{\text{elastic incr}}}}}} \end{bmatrix}$ where i ranges over the number of antisymmetric active control surfaces, $i=1, n-2$
(CONTRL)–S	$\begin{bmatrix} C_{L_{\delta_{e_{\text{rigid}}}}} & C_{L_{\delta_{e_{\text{elastic incr}}}}} & C_{L_{\delta_{c_{i_{\text{rigid}}}}}} & C_{L_{\delta_{c_{i_{\text{elastic incr}}}}}} \\ C_{D_{\delta_{e_{\text{rigid}}}}} & C_{D_{\delta_{e_{\text{elastic incr}}}}} & C_{D_{\delta_{c_{i_{\text{rigid}}}}}} & C_{D_{\delta_{c_{i_{\text{elastic incr}}}}}} \\ C_{m_{\delta_{e_{\text{rigid}}}}} & C_{m_{\delta_{e_{\text{elastic incr}}}}} & C_{m_{\delta_{c_{i_{\text{rigid}}}}}} & C_{m_{\delta_{c_{i_{\text{elastic incr}}}}}} \end{bmatrix}$ where i ranges over the number of symmetric active controls, $i=1, n-1$
(CSNAMES)A	$\begin{bmatrix} \text{AILERON} & \text{RUDDER} & AC_{\text{name}_i} \cdots\cdots AC_{\text{name}_n} \end{bmatrix}$ where i ranges over the number of antisymmetric active controls, $i=1, n-2$
(CSNAMES)S	$\begin{bmatrix} \text{ELEVATOR} & AC_{\text{name}_i} \cdots\cdots AC_{\text{name}_n} \end{bmatrix}$ where i ranges over the number of symmetric active controls, $i=1, n-1$

Figure 11. — General Form of Derivative Matrices on Input File (SDSSTP)

FLEXSTAB name	Engineering symbol
(STATIC)–S	$\begin{bmatrix} C_{L_{1_{\text{rigid}}}} & C_{L_{1_{\text{elastic incr}}}} & C_{L_{\hat{U}_{\text{rigid}}}} & C_{L_{\hat{U}_{\text{elastic incr}}}} & C_{L_{\alpha_{\text{rigid}}}} & C_{L_{\alpha_{\text{elastic incr}}}} & C_{L_{\hat{q}_{\text{rigid}}}} & C_{L_{\hat{q}_{\text{elastic incr}}}} \\ 0 & 0 & C_{D_{\hat{U}_{\text{rigid}}}} & C_{D_{\hat{U}_{\text{elastic incr}}}} & C_{D_{\alpha_{\text{rigid}}}} & C_{D_{\alpha_{\text{elastic incr.}}}} & C_{D_{\hat{q}_{\text{rigid}}}} & C_{D_{\hat{q}_{\text{elastic incr}}}} \\ 0 & 0 & C_{m_{\hat{U}_{\text{rigid}}}} & C_{m_{\hat{U}_{\text{elastic incr}}}} & C_{m_{\alpha_{\text{rigid}}}} & C_{m_{\alpha_{\text{elastic incr.}}}} & C_{m_{\hat{q}_{\text{rigid}}}} & C_{m_{\hat{q}_{\text{elastic incr}}}} \end{bmatrix}$
(STATIC)–A	$\begin{bmatrix} C_{Y_{\beta_{\text{rigid}}}} & C_{Y_{\beta_{\text{elastic incr}}}} & C_{Y_{\hat{p}_{\text{rigid}}}} & C_{Y_{\hat{p}_{\text{elastic incr}}}} & C_{Y_{\hat{r}_{\text{rigid}}}} & C_{Y_{\hat{r}_{\text{elastic incr}}}} \\ C_{\ell_{\beta_{\text{rigid}}}} & C_{\ell_{\beta_{\text{elastic incr}}}} & C_{\ell_{\hat{p}_{\text{rigid}}}} & C_{\ell_{\hat{p}_{\text{elastic incr}}}} & C_{\ell_{\hat{r}_{\text{rigid}}}} & C_{\ell_{\hat{r}_{\text{elastic incr}}}} \\ C_{n_{\beta_{\text{rigid}}}} & C_{n_{\beta_{\text{elastic incr}}}} & C_{n_{\hat{p}_{\text{rigid}}}} & C_{n_{\hat{p}_{\text{elastic incr}}}} & C_{n_{\hat{r}_{\text{rigid}}}} & C_{n_{\hat{r}_{\text{elastic incr}}}} \end{bmatrix}$

Figure 11. – (Concluded)

The files of the modified equations of motion and loads equations have the same format as the input formats shown in figures 8 and 9 except that the size of the modified matrices may be larger by:

$$NDOF = NDOFI + NDOF_{NSEN} + NDOF_{NSAS}$$

where:

NDOF	= The total number of degrees of freedom
NDOFI	= The number of degrees of freedom input
$NDOF_{NSEN}$	= The sensors number of degrees of freedom
$NDOF_{NSAS}$	= The SAS number degrees of freedom

The default name is EQEOM for the modified equations of motion file and EQLOD for the modified load equations file.

The QR equations output magnetic file format is shown in figure 12.

All matrices contained on these output magnetic files are written by the WRTETP subroutine.

6.6 RESTRICTIONS

The following restrictions apply only when generating matrices for QR. These restrictions are due to the limitation of the QR program (ref. 6).

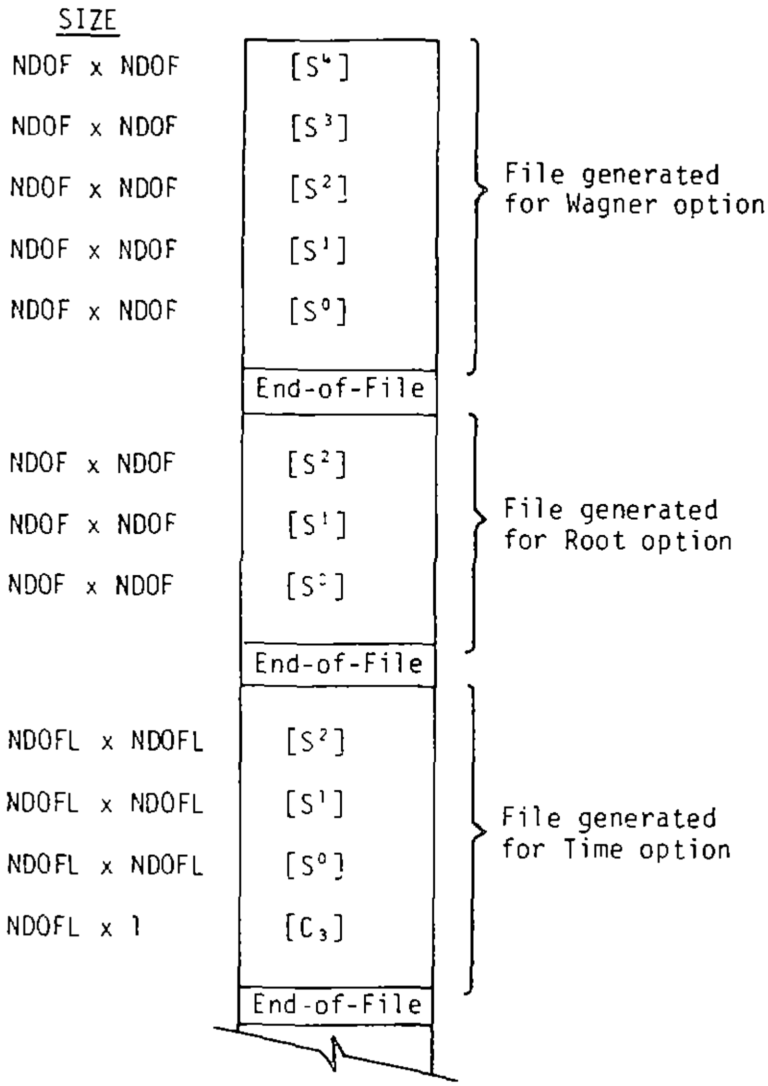
- No gradual penetration is allowed when forming matrices for the time history solution in QR.
- No multi-forcing-function is allowed for a QR time history solution.
- Only first frequency used (EQMOD restriction).
- Only one load set used (specified via card).

6.7 DIAGNOSTICS

6.7.1 FATAL ERRORS

All fatal errors detected by L219 (EQMOD) will result in the printing of a diagnostic error message. These messages are self-explanatory and are of the following format:

    ******** FATAL ERROR (nnnnn) DIAGNOSED WHILE EXECUTING ROUTINE
             (routine name).
             Any additional error message follows.



where:

NDOF - Number of degrees of freedom

NDOFL - NDOF + NLD

NLD - Number of loads

Figure 12. — QR Equations Output File (QRTAP)

where nnnnn is a diagnostic number from the following list:

Error number	Description
1	$EQMOD is not the first card input data
2	Error returned from FETAD (see FETAD error message)
3	Keyword (aaaaaaaaaa) with code number (nnnnn) is not recognized.
4	Total number of degrees of freedom (nnnnn) not within range of 1 and 100.
5	Total number of panels (nnnnn) not within range of 0 and 50.
6	Number of frequencies (nnnnn) not within range of 0 and 20.
7	Output file name (aaaaaaaaaa) for modified EOM matrices, invalid.
8	Output file name (aaaaaaaaaa) for modified LOADS matrices, invalid.
9	The requested number of (LOADS/EOM/QR) sets (nnnnn + 1) is greater than the maximum allowed (nnnnn).
10	READTP error number (nnnnn) returned. (See section 6.7.3.)
11	Dimension on first matrix of DYLOFLEX (EOM/LOADS) input tape is not 30 x 1.
12	(EOM/LOADS) scale data must be grouped together.
13	(EOM/LOADS) replacement data must be grouped together.
14	(EOM/LOADS) increment data must be grouped together.
15	Body axis data already defined.
16	Keyword (aaaaaaaaaa) is not recognized as a SENSOR data.
17	Sensor matrix (aaaaa) not grouped in increasing (M1BAR,M2BAR, M3BAR) order.
18	(keyword) matrix name (aaaaa) not recognized; (keyword) matrix name (aaaaa) and frequency number (nnnnn) must be grouped together.
19	QR option not specified; must have one of the following (WAGNER, ROOT,TIME).
20	Field length requested is too small for this problem.
21	WRTETP error number (nnnnn) returned. (See section 6.7.4.)
22	Matrix (aaaaaaaaaa) frequency (nnnnn) is greater than number of frequencies (nnnnn).
23	Derivatives input; VT, B, or CBAR contain zero.
24	Error returned by FETCHM while reading matrix (aaaaaaaaaa).
25	Control surface name specified (aaaaaaaaaa) does not equal to names read from SDSSTP (list of names follow).
26	QR time option; PHI-TILDA OR PHI-TILDA-BAR (aaaaaaaaaa) has more than one column.

6.7.2 WARNING MESSAGES

All warning messages will be self- explanatory and printed in the following format:

******** WARNING MESSAGE (nnnnn) DIAGNOSED WHILE EXECUTING ROUTINE (routine name).
(Any additional warning message follows.)

where nnnnn is a warning message number from the following list.

Warning number	Description
1	The maximum number of title cards (nnnnn) has already been read. The above title card is treated as a comment.
2	End of record encountered before $QUIT card. $QUIT card assumed.

6.7.3 READTP ERROR CODES

Code	Description
0	No errors detected.
2	File or matrix skipping code is negative.
3	Dimensioned row size ≤ 0.
4	Dimensioned row size is less than the actual row size.
5	Name specified does not match the matrix name on the file.
6	NROWS*NCOLS ≤ 0 or greater than the buffer limit of 10000.
7	End-of-file read instead of the two-record matrix.
1000+I	Error discovered trying to skip files. I files remained to be skipped when an end-of- information (EOI) was encountered.
3000+I	Error discovered trying to skip matrices. I/2 matrices remained to be skipped when an end-of-file (EOF) was encountered.

6.7.4 WRTETP ERROR CODES

Code	Description
0	No error detected.
2	File on matrix skipping code is negative.
3	Dimensioned row size ≤ 0.
4	Dimensioned row size is less than the actual row size.
6	NROWS*NCOLS ≤ 0.
1000+I	End-of-information (EOI) was read trying to skip files. I files remained to be skipped.
3000+I	End-of-file (EOF) was read trying to skip matrices. I/2 matrices remained to be skipped.

7.0 SAMPLE PROBLEM

The sample problem in this section consists of three small test cases (numbers 2, 3 and 5) formed to exercise most of the options available in the program. The size of these test cases allowed them to be easily checked by hand calculations.

Test Case 2:

This sample problem assumes that the matrix coefficients for the equations of motion were generated by L217 (EOM) for a six degree of freedom system. The load equation coefficients are assumed to be read from a file LODTAP that was not generated by the DYLOFLEX system. Modifications to the equations include overwriting the rigid body terms in the equations of motion with stability derivative data, adding sensor equations to the system, and replacing and incrementing matrix elements. Wagner indical lift growth effects shall be included, and a data tape for QR rooting and time history analysis shall be generated.

Test Case 3:

This sample problem assumes all load and equations of motion coefficient matrices were generated by DYLOFLEX programs for a four degree of freedom system. Modifications include a scaler multiplication of M_4, a transformation of the equations into the body-fixed axes, and replacement and incrementation of certain load matrix coefficients.

Test Case 5:

The last sample problem deals with a seven degree of freedom system. Here the equations of motion matrix coefficients will be modified by overwriting the rigid body elements with stability derivative data. Sensor and stability augmentation equations will also be added.

Boeing Commercial Airplane Company
P.O. Box 3707
Seattle, Washington 98124
May 1977

```
$EQMOD      TEST CASE 2                                                          1.0
TITLE       CHECK CASE 2, MODIFY DERIVATIVES FROM CARDS, ADD SENSORS, QR TP
C           6-DEGREES OF FREEDOM , 2-FREQUENCIES                                 2.1A
SIZE           6    0    2                                                       3.0
OUTPUT      EQEOM2         1       EQLOD2          1                             4.1
PRINT       INPUT      MATRIX      FREQUENCY       2                             4.2
PRINT       OUTPUT     MATRICES    CHANGED                                       4.3
SYMMETRIC                                                                        5.0
C           EQ. OF MOTION ASSUMED TO HAVE BEEN GENERATED BY  L217(EOM)           2.1B
$EOM        EOMTAP          2DYLOFLX                                             6.0
C           ONE CONTROL SURFACE AND USE UNSTEADY DERIVATIVES                     2.1C
DERIVATIVES FROM      CARD            1     1 2.0       10.0                     7.1
         1    2    4                                                             7.2
                      .1          2.5        1.0        3.0                      7.3
4.0         1.0       1.0         7.0        1.0        10.0                     7.5.1R
1.0         .1        -.1         -1.0       1.0        -2.0                     7.5.2E
2.0         3.0       7.0                                                        7.6.1R
.1          -1.0      1.0                                                        7.6.2E
4.0         1.0       6.0                                                        7.7.1R
-2.0        -1.0      -1.0                                                       7.7.2E
2.0         3.0       5.0                                                        7.8.1R
0.0         0.0       0.0                                                        7.8.2E
C           SENSOR DATA FROM L218(LOADS)
SENSOR      LODTP2         2     3DYLOFLX                                        9.1
M3BAR          1    5    2    6                                                  9.2
END SENSOR                                                                       9.3
REPLACE     M1                        3     3 200.                               11.0
INCREMENT   M3                        3     3 10.                                12.0-1
INCREMENT   M3                        3     4  5.                                12.0-2
C           LOAD TAPE GENERATED OUTSIDE OF DYLOFLEX SYSTEM                       2.1E
$LOADS      LODTAP         1    2    0    0    3    4    5    8    0             14.0
SCALE       M4BAR          2     2.0                                             15.0
$QR         QRTAP          1                                                     18.1A
QREOM       EQEOM2          DYLOFLX                                              18.2A
ROOT                                                                             18.4
$QR         QRTAP          2                                                     18.1B
QREOM       EQEOM2          DYLOFLX                                              18.2B
TIME                                                                             18.5
QRLOAD      EQLOD2                   DYLOFLX                                     18.6
$QR         QRTAP          3                                                     18.1C
QREOM       EQEOM2          DYLOFLX                                              18.2C
WAGNER      .5        .5          1.0        2.0                                 18.3
$QUIT                                                                            19.0
```

```
$EQMOD     TEST CASE 3                                                                          1.0
TITLE     CHECK CASE 3, MODIFY EOM AND LOADS
SIZE           4          2                                                                     3.0
OUTPUT     TEST4          1     DUMMY        1                                                  4.1
PRINT     OUTPUT     MATRICES  ALL                                                              4.3
SYMMETRIC                                                                                       5.0
$EOM      EOMTAP          2DYLOFLX                                                              6.0
SCALE     M4                    2.0                                                             8.0
BODY AXIS       1     2   3     1.0        10.0                                                 13.0
$LOADS    LODTAP          2    2DYLOFLX                                                         14.0
REPLACE   M38AR                       1    4 10.0                                               16.0-1
REPLACE   M4BAR           2           1    4  5.0                                               16.0-2
INCREMENT M38AR                       2    4  3.0                                               17.0
$QUIT                                                                                           19.0
```

```
$EQMOD     TEST CASE 5                                                            1.0
TITLE      CHECK CASE 5, MODIFY DERIVATIVES FROM CARDS AND ADD SAS AND SENSORS
SIZE             7         2                                                      3.0
OUTPUT     EQEOM           1    EQLOD       1                                     4-1
PRINT      INPUT     MATRIX     ALL                                               4.2
PRINT      OUTPUT    MATRIX     ALL                                               4.3
SYMMETRIC                                                                         5.0
$EOM       EOMTAP          1    1    0    1    1    1    1    0    0             6.0
DERIVATIVE FROM      CARDS           1    0 2.0       10.0                        7.1
         1    2    4                                                              7.2
                     .1         2.5       1.0       3.0       .2                  7.3
4.0        1.0       1.0        7.0       1.0       10.0                          7.5.1R
1.0        .1        -.1        -1.0      1.0       -2.0                          7.5.2E
2.0        3.0       7.0                                                          7.6.1R
.1         -1.0      1.0                                                          7.6.2E
4.0        1.0       6.0                                                          7.7.1R
-2.0       -1.0      -1.0                                                         7.7.2E
SENSOR                          3    0    1    1                                  9.1
M2BAR           3    5                                                            9.2-1
M3BAR           1    6                                                            9.2-2
END SENSOR                                                                        9.3
SAS                                                                               10.1
    7    7 1.0                                                                    10.2-1
    7    5           -1.0                                                         10.2-2
    7    6                      2.0                                               10.2-3
    4    7 1.0       2.0        3.0                                               10.2-4
END SAS                                                                           10.3
$QUIT
```

```
*****************************************************
*                                                   *
*   PROGRAM L219A2  VERSION JUNE 29,77 NOW RUNNING. *
*   THE PROGRAM IS PART OF THE DYLOFLX SYSTEM       *
*   DEVELOPED FOR NASA UNDER CONTRACT NAS1-13918.   *
*   DATE OF RUN IS  77/11/09.                       *
*   TIME OF RUN IS  09.46.18.                       *
*                                                   *
*****************************************************
  ($EOMOD    TEST CASE 2                                                    1.0     )
  (TITLE     CHECK CASE 2, MODIFY DERIVATIVES FROM CARDS, ADD SENSORS, QR TP        )
  (C         6-DEGREES OF FREEDOM , 2-FREQUENCIES                           2.1A    )
  (SIZE          6     0     2                                              3.0     )

        P R O B L E M   S I Z E
        NDOF   =     6, TOTAL NUMBER OF DEGREES OF FREEDOM.
        NPAN   =     0, TOTAL NUMBER OF PANELS.
        NFREQM =     2, NUMBER OF FREQUENCIES.
 (OUTPUT    EQEOM2          1      EQLOD2          1                        4.1     )

        O U T P U T   T A P E S
        IUTEOM = EQEOM2     , TAPE NAME FOR MODIFIED EOM MATRICES.
        IFLEOM =          1, FILE POSITION NUMBER OF IUTEOM.
        IUTLOD = EQLOD2     , TAPE NAME FOR MODIFIED LOADS MATRICES.
        IFLLOD =          1, FILE POSITION NUMBER OF IUTLOD.
 (PRINT     INPUT     MATRIX     FREQUENCY      2                           4.2     )

        INPUT MATRICES   P R I N T   OPTION.
        INPR =     2, IF INPR = -999, PRINT ALL INPUT MATRICES.
                               = 0, NO INPUT MATRICES PRINTED.
                               = N, MATRICES OF NTH FREQUENCY ONLY PRINTED.
 (PRINT     OUTPUT    MATRICES  CHANGED                                     4.3     )

        OUTPUT MATRICES   P R I N T   OPTION.
        IUTPR =  999, IF IUTPR = -999, PRINT ALL OUTPUT MATRICES.
                                 = 0, NO OUTPUT MATRICES PRINTED.
                                 = N, MATRICES OF NTH FREQUENCY ONLY PRINTED.
                                 = 999, PRINT ONLY MODIFIED MATRICES.
 (SYMMETRIC                                                                 5.0     )
 (C         EQ. OF MOTION ASSUMED TO HAVE BEEN GENERATED BY  L217(EOM)      2.1B    )
 ($EOM      EOMTAP          2DYLOFLX                                        6.0     )

        E Q U A T I O N S   O F   M O T I O N

        INEOM  = EOMTAP     , EOM INPUT TAPE NAME.
        INEOMF =          2, FILE POSITION NUMBER OF INEOM.

 (C         ONE CONTROL SURFACE AND USE UNSTEADY DERIVATIVES                2.1C    )
 (DERIVATIVES FROM    CARD           1     1 2.0       10.0                 7.1     )

        DERIVATIVES FOR SYMMETRIC ANALYSIS.
        IVOL   =       CARD, INPUT VOLUME
        NCS    =          1, NUMBER OF CONTROL SURFACE
        INDUN  =          1, UNSTEADY DERIVATIVE INDICATOR
        QUEBAR =  .200E+01, DYNAMIC PRESSURE
        VT     =  .100E+02, VELOCITY (TRUE AIR SPEED)

        COLUMN NUMBERS OF RIGID BODY FREEDOMS
        IXCOL  =          0, COLUMN OF X FREEDOMS
        IZCOL  =          1, COLUMN OF Z FREEDOMS
        ITCOL  =          2, COLUMN OF THETA FREEDOMS
        COLUMN OF DELTA CONTROL SURFACE FREEDOMS FOLLOW
            4
```

```
        CONSTANTS ASSOCIATED WITH DERIVATIVES
        XREF    = 0.         , X-COORDINATE OF MOMENT REFERENCE
        ZREF    = C.         , Z-COORDINATE OF MOMENT REFERENCE
        ALPHA1 =   .175E-02, IG ANGLE OF ATTACK
        SW      =  .250E+C1, WING REFERENCE AREA
        CBAR    =  .100E+01, REFERENCE CHORD
        CL1R    =  .3C0E+01, RIGID STEADY STATE DERIVATIVE
        CL1E    = 0.         , ELASTIC STEADY STATE DERIVATIVE

        STEADY STATE DERIVATIVES FROM CARD
        CLU     =  .400E+01, C-L-UHAT-RIGID
        CDU     =  .100E+01, C-D-UHAT-RIGID
        CMUREF  =  .100E+01, C-M-UHAT-RIGID-REF
        CLA     =  .700E+01, C-L-ALPHA-RIGID
        CDA     =  .100E+01, C-D-ALPHA-RIGID
        CMAREF  =  .100E+02, C-M-ALPHA-RIGID-REF

        CLUE    =  .1C0E+01, C-L-UHAT-ELASTIC
        CDUE    =  .100E+00, C-D-UHAT-ELASTIC
        CLAE    = -.100E+01, C-L-ALPHA-ELASTIC
        CDAE    =  .100E+01, C-D-ALPHA-ELASTIC

        CLQ     =  .200E+01, C-L-QUEHAT-RIGID
        CDQ     =  .300E+01, C-D-QUEHAT-RIGID
        CMQREF  =  .700E+01, C-M-QUEHAT-RIGID-REF

        CLQE    =  .100E+00, C-L-QHAT-ELASTIC
        CDQE    = -.100E+01, C-D-QHAT-ELASTIC

        CONTROL SURFACE DERIVATIVES
        CLD (C-L-DELTA-RIGID) FOLLOWS
          .400E+01
        CDD (C-D-DELTA-RIGID) FOLLOWS
          .100E+01
        CMDREF (C-M-DELTA-REF) FOLLOWS
          .600E+01
        CLDE (C-L-DELTA-ELASTIC) FOLLOWS
         -.20CE+01
        CDDE (C-C-DELTA-ELASTIC) FOLLOWS
         -.100E+01

UNSTEADY DERIVATIVES
        CLADOT =  .200E+01, C-L-ALPHA-DOT-HAT-RIGID
        CDADOT =  .300E+01, C-D-ALPHA-DOT-HAT-RIGID
        CMADRF =  .500E+01, C-M-ALPHA-DOT-HAT-RIGID-REF

        CLADTE = 0.         , C-L-ALPHA-DOT-HAT-ELASTIC
        CDADTE = 0.         , C-D-ALPHA-DOT-HAT-ELASTIC
  (C         SENSOR DATA FROM L218(LOADS)                                          )
  (SENSOR    LODTP2        2    3DYLOFLX                             9.1           )

        INSEN  = LODTP2     , SENSORS INPUT TAPE NAME.
        INSENF =            2, FILE POSITION NUMBER OF INSEN.
        NLDSEN =            3, NUMBER OF LOADS ON SENSOR TAPE.
   MATRIX         IN   OUT
  (M3BAR          1    5    2    6                                   9.2           )
  (END SENSOR                                                        9.3           )
  (REPLACE   M1                       3    3 200.                    11.0          )
  (INCREMENT M3                       3    3 10.                     12.0-1        )
  (INCREMENT M3                       3    4  5.                     12.0-2        )
  (C         LOAD TAPE GENERATED OUTSIDE OF DYLOFLEX SYSTEM          2.1E          )
  ($LOADS    LODTAP        1    2    0    0    3    4    5    8    0   14.0        )

        L O A D S   E Q U A T I O N
```

```
        INLOD  = LODTAP      , LOADS INPUT TAPE NAME.
        INLODF =            1, FILE POSITION NUMBER OF INLOD.
        NLDOU  =            2, NUMBER OF OUTPUT LOADS.

  (SCALE     M4BAR          2       2.0                                                 15.0     )
  ($QR       QRTAP          1                                                           18.1A    )

        Q R

        IUTQR  = QRTAP       , TAPE NAME FOR QR MATRICES.
        IFLQR  =            1, FILE POSITION NUMBER OF IUTQR.

  (QRECM     EQEOM2           DYLOFLX                                                   18.2A    )
  (ROOT                                                                                 18.4     )

Q R   R O O T   O P T I O N.
  ($QR       QRTAP          2                                                           18.1B    )

        Q R

        IUTQR  = QRTAP       , TAPE NAME FOR QR MATRICES.
        IFLQR  =            2, FILE POSITION NUMBER OF IUTQR.

  (QRECM     EQEOM2           DYLOFLX                                                   18.2B    )
  (TIME                                                                                 18.5     )

Q R   T I M E   O P T I O N.
  (QRLOAD    EQLOD2                  DYLOFLX                                            18.6     )
  ($QR       QRTAP          3                                                           18.1C    )

        Q R

        IUTQR  = QRTAP       , TAPE NAME FOR QR MATRICES.
        IFLQR  =            3, FILE POSITION NUMBER OF IUTQR.

  (QRECM     EQEOM2           DYLOFLX                                                   18.2C    )
  (WAGNER    .5         .5          1.0         2.0                                     18.3     )

Q R   W A G N E R   O P T I O N.
        AQR  =  .500E+00, WAGENR FUNCTION, A.
        BQR  =  .500E+00, WAGENR FUNCTION, B.
        ALQR =  .100E+01, WAGENR FUNCTION, ALPHA.
        BEQR =  .200E+01, WAGNER FUNCTION, BETA.
  ($QUIT                                                                                19.0     )
```

E O M   M A T R I X   E Q U A T I O N S

TITLE   CHECK CASE 2, MODIFY DERIVATIVES FROM CARDS, ADD SENSORS, OR TP

I N P U T   M A T R I C E S   FREQUENCY   1

S E N S O R

--------- MATRIX   M2BAR   DIMENSIONED (3X   4)

ROW 1	0.	0.	0.	0.
ROW 2	0.	0.	0.	0.
ROW 3	.1000E+01	-.1000E+02	.3000E+01	0.

--------- MATRIX   M3BAR   DIMENSIONED (3X   4)

ROW 1	.1000E+01	.1200E+02	.8000E+00	0.
ROW 2	.1000E+01	-.6000E+01	.5000E+00	0.
ROW 3	0.	0.	0.	0.

O U T P U T   M A T R I C E S   FREQUENCY   1

--------- MATRIX   M1   DIMENSIONED (6X   6)

ROW 1	0.	0.	0.	0.	0.	0.
ROW 2	0.	0.	0.	0.	0.	0.
ROW 3	0.	0.	.2000E+03	0.	0.	0.
ROW 4	0.	0.	0.	0.	0.	0.
ROW 5	0.	0.	0.	0.	-.1000E+01	0.
ROW 6	0.	0.	0.	0.	0.	-.1000E+01

--------- MATRIX   M3   DIMENSIONED (6X   6)

ROW 1	.1000E+04	0.	0.	0.	0.	0.
ROW 2	0.	.4000E+05	0.	0.	0.	0.
ROW 3	0.	0.	.2000E+02	.5000E+01	0.	0.
ROW 4	0.	0.	0.	.1500E+02	0.	0.
ROW 5	.1000E+01	.1200E+02	.8000E+00	0.	0.	0.
ROW 6	.1000E+01	-.6000E+01	.5000E+00	0.	0.	0.

--------- MATRIX   FREQM   DIMENSIONED (2X   1)

ROW 1	0.
ROW 2	.2000E+01

INPUT MATRICES FREQUENCY 1

OUTPUT MATRICES FREQUENCY 1

--------- MATRIX M4 DIMENSIONED (6X 6)

ROW	1	0.	.3497E+02	.6000E+01	.2001E+02	0.	0.
ROW	2	0.	-.4999E+02	.8000E+01	-.3000E+02	0.	0.
ROW	3	0.	.4000E+01	.2000E+01	-.1000E+01	0.	0.
ROW	4	0.	.1000E+01	-.1000E+01	.4000E+01	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.

--------- MATRIX M5 DIMENSIONED (6X 6)

ROW	1	.3501E+01	.1003E+01	.7000E+01	.3000E+01	0.	0.
ROW	2	-.5000E+01	-.3000E+01	.4000E+01	.2000E+01	0.	0.
ROW	3	-.4000E+00	.1000E+01	.2000E+01	.1000E+01	0.	0.
ROW	4	-.1000E+00	.1000E+01	.1000E+01	.2000E+01	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.

I N P U T   M A T R I C E S   FREQUENCY   2

--------- MATRIX M4 DIMENSIONED (4X 4)

ROW	1	0.	-.3000E+01	.7000E+01	.1000E+01
ROW	2	0.	.6000E+01	.9000E+01	.1000E+01
ROW	3	0.	.5000E+01	.3000E+01	-.1000E+01
ROW	4	0.	.2000E+01	-.1000E+01	.3000E+01

--------- MATRIX M5 DIMENSIONED (4X 4)

ROW	1	.3000E+00	.6000E+01	.6000E+01	.4000E+01
ROW	2	-.6000E+00	.5000E+01	.3000E+01	.3000E+01
ROW	3	-.5000E+00	.2000E+01	.1000E+01	.2000E+01
ROW	4	-.2000E+00	.2000E+01	0.	.2000E+01

--------- MATRIX C3 DIMENSIONED (8X 1)

ROW	1	.3000E+00
ROW	2	.1000E+00
ROW	3	-.6000E+00
ROW	4	.2000E+00
ROW	5	-.5000E+00
ROW	6	.1000E+00
ROW	7	-.2000E+00
ROW	8	.3000E+00

O U T P U T   M A T R I C E S   FREQUENCY   2

--------- MATRIX M4 DIMENSIONED (6X 6)

ROW	1	0.	.3497E+02	.7000E+01	.2001E+02	0.	0.
ROW	2	0.	-.4999E+02	.9000E+01	-.3000E+02	0.	0.
ROW	3	0.	.5000E+01	.3000E+01	-.1000E+01	0.	0.
ROW	4	0.	.2000E+01	-.1000E+01	.3000E+01	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.

--------- MATRIX M5 DIMENSIONED (6X 6)

ROW	1	.3501E+01	.1003E+01	.6000E+01	.4000E+01	0.	0.
ROW	2	-.5000E+01	-.3000E+01	.3000E+01	.3000E+01	0.	0.
ROW	3	-.5000E+00	.2000E+01	.1000E+01	.2000E+01	0.	0.
ROW	4	-.2000E+00	.2000E+01	0.	.2000E+01	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.

L O A D S   M A T R I X   E Q U A T I O N S

TITLE   CHECK CASE 2, MODIFY DERIVATIVES FROM CARDS, ADD SENSORS, QR TP

I N P U T   M A T R I C E S   FREQUENCY   1

O U T P U T   M A T R I C E S   FREQUENCY   1

I N P U T   M A T R I C E S   FREQUENCY   1

O U T P U T   M A T R I C E S   FREQUENCY   1

---------- MATRIX   M4BAR   DIMENSIONED (2X   6)

ROW 1	.8000E+01	.4000E+01	.3000E+01	0.	0.	0.
ROW 2	.6000E+01	.3000E+01	.5000E+01	.2000E+01	0.	0.

I N P U T   M A T R I C E S   FREQUENCY   2

---------- MATRIX   M4BAR   DIMENSIONED (2X   4)

ROW	1	.7000E+01	.3000E+01	.2000E+01	-.1000E+01
ROW	2	.5000E+01	.2000E+01	.4000E+01	.1000E+01

---------- MATRIX   M5BAR   DIMENSIONED (2X   4)

ROW	1	-.30C0E+C1	0.	-.2000E+01	.4000E+01
ROW	2	.2000E+01	0.	-.3000E+01	-.1000E+01

---------- MATRIX   C3BAR   DIMENSIONED (4X   1)

ROW	1	-.3000E+01
ROW	2	.1000E+C0
ROW	3	.20C0E+C1
ROW	4	.20C0E+00

O U T P U T   M A T R I C E S   FREQUENCY   2

---------- MATRIX   M4BAR   DIMENSIONED (2X   6)

ROW	1	.1400E+02	.6000E+01	.4000E+01	-.2000E+01	0.	0.
ROW	2	.1000E+02	.4000E+01	.8000E+01	.2000E+01	0.	0.

Q R   M A T R I C E S   G E N E R A T E D

ROOT OPTION

TITLE   CHECK CASE 2, MODIFY DERIVATIVES FROM CARDS, ADD SENSORS, QR TP

--------- MATRIX S**2 DIMENSIONED (6X 6)

ROW	1	.1000E+04	0.	0.	0.	0.	0.
ROW	2	0.	.4000E+05	0.	0.	0.	0.
ROW	3	0.	0.	.2000E+02	.5000E+01	0.	0.
ROW	4	0.	0.	0.	.1500E+02	0.	0.
ROW	5	.1000E+01	.1200E+02	.8000E+00	0.	0.	0.
ROW	6	.1000E+01	-.6000E+01	.5000E+00	0.	0.	0.

--------- MATRIX S**1 DIMENSIONED (6X 6)

ROW	1	.3501E+01	.1003E+01	.7000E+01	.3000E+01	0.	0.
ROW	2	-.5000E+01	-.3000E+01	.4000E+01	.2000E+01	0.	0.
ROW	3	-.4000E+00	.1000E+01	.2000E+01	.1000E+01	0.	0.
ROW	4	-.1000E+00	.1000E+01	.1000E+01	.2000E+01	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.

--------- MATRIX S**0 DIMENSIONED (6X 6)

ROW	1	0.	.3497E+02	.6000E+01	.2001E+02	0.	0.
ROW	2	0.	-.4999E+02	.8000E+01	-.3000E+02	0.	0.
ROW	3	0.	.4000E+01	.2020E+03	-.1000E+01	0.	0.
ROW	4	0.	.1000E+01	-.1000E+01	.4000E+01	0.	0.
ROW	5	0.	0.	0.	0.	-.1000E+01	0.
ROW	6	0.	0.	0.	0.	0.	-.1000E+01

Q R   M A T R I C E S   G E N E R A T E D

TIME SOLUTION OPTION

TITLE   CHECK CASE 2, MODIFY DERIVATIVES FROM CARDS, ADD SENSORS, QR TP

--------- MATRIX   S**2   DIMENSIONED (8X   8)

ROW 1	.1000E+04	0.	0.	0.	0.	0.	0.	0.
ROW 2	0.	.4000E+05	0.	0.	0.	0.	0.	0.
ROW 3	0.	0.	.2000E+02	.5000E+01	0.	0.	0.	0.
ROW 4	0.	0.	0.	.1500E+02	0.	0.	0.	0.
ROW 5	.1000E+01	.1200E+02	.8000E+00	0.	0.	0.	0.	0.
ROW 6	.1000E+01	-.6000E+01	.5000E+00	0.	0.	0.	0.	0.
ROW 7	.7000E+01	.8000E+01	.4000E+01	.1000E+01	0.	0.	0.	0.
ROW 8	.6000E+01	.3000E+01	-.1000E+01	.2000E+01	0.	0.	0.	0.

--------- MATRIX   S**1   DIMENSIONED (8X   8)

ROW 1	.3501E+01	.1003E+01	.7000E+01	.3000E+01	0.	0.	0.	0.
ROW 2	-.5000E+01	-.3000E+01	.4000E+01	.2000E+01	0.	0.	0.	0.
ROW 3	-.4000E+00	.1000E+01	.2000E+01	.1000E+01	0.	0.	0.	0.
ROW 4	-.1000E+00	.1000E+01	.1000E+01	.2000E+01	0.	0.	0.	0.
ROW 5	0.	0.	0.	0.	0.	0.	0.	0.
ROW 6	0.	0.	0.	0.	0.	0.	0.	0.
ROW 7	-.2000E+01	.1000E+01	-.1000E+01	.5000E+01	0.	0.	0.	0.
ROW 8	.3000E+01	-.1000E+01	-.2000E+01	-.2000E+01	0.	0.	0.	0.

--------- MATRIX   S**0   DIMENSIONED (8X   8)

ROW 1	0.	.3497E+02	.6000E+01	.2001E+02	0.	0.	0.	0.
ROW 2	0.	-.4999E+02	.8000E+01	-.3000E+02	0.	0.	0.	0.
ROW 3	0.	.4000E+01	.2020E+03	-.1000E+01	0.	0.	0.	0.
ROW 4	0.	.1000E+01	-.1000E+01	.4000E+01	0.	0.	0.	0.
ROW 5	0.	0.	0.	0.	-.1000E+01	0.	0.	0.
ROW 6	0.	0.	0.	0.	0.	-.1000E+01	0.	0.
ROW 7	.8000E+01	.4000E+01	.3000E+01	0.	0.	0.	-.1000E+01	0.
ROW 8	.6000E+01	.3000E+01	.5000E+01	.2000E+01	0.	0.	0.	-.1000E+01

--------- MATRIX   VECTOR   DIMENSIONED (8X   1)

ROW 1	-.3501E+01
ROW 2	.5000E+01
ROW 3	-.4000E+00
ROW 4	-.1000E+00
ROW 5	0.
ROW 6	0.
ROW 7	-.2000E+01
ROW 8	.3000E+01

Q R   M A T R I C E S   G E N E R A T E D

WAGNER FUNCTION OPTION

TITLE   CHECK CASE 2, MODIFY DERIVATIVES FROM CARDS, ADD SENSORS, QR TP

--------- MATRIX S**4 DIMENSIONED (6X 6)

ROW	1	.1000E+04	0.	0.	0.	0.	0.
ROW	2	0.	.4000E+05	0.	0.	0.	0.
ROW	3	0.	0.	.2000E+02	.5000E+01	0.	0.
ROW	4	0.	0.	0.	.1500E+02	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.

--------- MATRIX S**3 DIMENSIONED (6X 6)

ROW	1	.3000E+C4	0.	0.	0.	0.	0.
ROW	2	0.	.1200E+06	0.	0.	0.	0.
ROW	3	0.	0.	.6000E+02	.1500E+02	0.	0.
ROW	4	0.	0.	0.	.4500E+02	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	C.	0.	0.	0.

--------- MATRIX S**2 DIMENSIONED (6X 6)

ROW	1	.2005E+04	.1504E+01	.1050E+02	.4500E+01	0.	0.
ROW	2	-.7500E+01	.8000E+05	.6000E+01	.3000E+01	0.	0.
ROW	3	-.6000E+00	.1500E+01	.2430E+03	.1150E+02	0.	0.
ROW	4	-.1500E+00	.1500E+01	.1500E+01	.3300E+02	0.	0.
ROW	5	.1000E+01	.1200E+02	.8000E+00	0.	0.	0.
ROW	6	.1000E+01	-.6000E+01	.5000E+00	0.	0.	0.

--------- MATRIX S**1 DIMENSIONED (6X 6)

ROW	1	.7002E+01	.5447E+02	.2300E+02	.3601E+02	0.	0.
ROW	2	-.1000E+02	-.8099E+02	.2000E+02	-.4100E+02	0.	0.
ROW	3	-.8000E+00	.8000E+01	.6070E+03	.5000E+00	0.	0.
ROW	4	-.2000E+00	.3500E+01	.5000E+00	.1000E+02	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.

--------- MATRIX S**0 DIMENSIONED (6X 6)

ROW	1	0.	.6995E+02	.1200E+02	.4002E+02	0.	0.
ROW	2	0.	-.9998E+02	.1600E+02	-.6000E+02	0.	0.
ROW	3	0.	.8000E+01	.4040E+03	-.2000E+01	0.	0.
ROW	4	0.	.2000E+01	-.2000E+01	.8000E+01	0.	0.
ROW	5	0.	0.	0.	0.	-.1000E+01	0.
ROW	6	0.	C.	0.	0.	0.	-.1000E+01

```
************************************************************
*                                                          *
*    PROGRAM L219A2  VERSION JUNE 29,77 IS FINISHED.       *
*    DATE OF RUN IS  77/11/09.                             *
*    TIME OF RUN IS  09.46.22.                             *
*                                                          *
************************************************************
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*****************************************************
*                                                   *
*   PROGRAM L219A2  VERSION JUNE 29,77 NOW RUNNING. *
*   THE PROGRAM IS PART OF THE DYLOFLX SYSTEM       *
*   DEVELOPED FOR NASA UNDER CONTRACT NAS1-13918.   *
*   DATE OF RUN IS  77/11/09.                       *
*   TIME OF RUN IS  09.46.23.                       *
*                                                   *
*****************************************************
  ($EQMOD    TEST CASE 3                                                  1.0     )
  (TITLE    CHECK CASE 3, MODIFY EOM AND LOADS                                    )
  (SIZE          4          2                                             3.0     )

        P R O B L E M   S I Z E
        NDOF   =     4, TOTAL NUMBER OF DEGREES OF FREEDOM.
        NPAN   =     0, TOTAL NUMBER OF PANELS.
        NFREQM =     2, NUMBER OF FREQUENCIES.
  (OUTPUT    TEST4         1      DUMMY          1                        4.1     )

        O U T P U T   T A P E S
        IUTEOM = TEST4      , TAPE NAME FOR MODIFIED EOM MATRICES.
        IFLEOM =           1, FILE POSITION NUMBER OF IUTEOM.
        IUTLOD = DUMMY      , TAPE NAME FOR MODIFIED LOADS MATRICES.
        IFLLOD =           1, FILE POSITION NUMBER OF IUTLOD.
  (PRINT     OUTPUT    MATRICES  ALL                                      4.3     )

        OUTPUT MATRICES   P R I N T   OPTION.
        IUTPR = -999, IF IUTPR = -999, PRINT ALL OUTPUT MATRICES.
                               = 0, NO OUTPUT MATRICES PRINTED.
                               = N, MATRICES OF NTH FREQUENCY ONLY PRINTED.
                               = 999, PRINT ONLY MODIFIED MATRICES.
  (SYMMETRIC                                                              5.0     )
  ($EOM      EOMTAP        2DYLOFLX                                       6.0     )

        E Q U A T I O N S   O F   M O T I O N

        INEOM  = EOMTAP     , EOM INPUT TAPE NAME.
        INEOMF =           2, FILE POSITION NUMBER OF INEOM.

  (SCALE     M4                  2.0                                      8.0     )
  (BODY AXIS     1    2    3     1.0        10.0                         13.0     )

        BODY AXIS SYMMETRIC
        ICOLX  =           1, X-     COLUMN OF ORIGINAL AXIS.
        ICOLZ  =           2, Z-     COLUMN OF ORIGINAL AXIS.
        ICOLT  =           3, THETA-COLUMN OF ORIGINAL AXIS.
        ALPHA1 =   .175E-01, ALPHA1, ANGLE OF ATTACK.
        BODYVT =   .100E+02, VT, VELOCITY - TRUE AIR SPEED.

  ($LOADS    LODTAP        2     2DYLOFLX                                14.0     )

        L O A D S   E Q U A T I O N

        INLOD  = LODTAP     , LOADS INPUT TAPE NAME.
        INLODF =           2, FILE POSITION NUMBER OF INLOD.
        NLDOU  =           2, NUMBER OF OUTPUT LOADS.

  (REPLACE   M3BAR                   1    4 10.0                         16.0-1   )
  (REPLACE   M4BAR         2         1    4  5.0                         16.0-2   )
  (INCREMENT M3BAR                   2    4  3.0                         17.0     )
  ($QUIT                                                                 19.0     )
```

E O M   M A T R I X   E Q U A T I O N S

TITLE   CHECK CASE 3, MODIFY EOM AND LOADS

O U T P U T   M A T R I C E S   FREQUENCY   1

--------- MATRIX M1 DIMENSIONED (4X 4)

ROW 1	0.	0.	0.	0.
ROW 2	0.	0.	0.	0.
ROW 3	0.	0.	.1000E+03	0.
ROW 4	0.	0.	0.	0.

--------- MATRIX M2 DIMENSIONED (4X 4)

ROW 1	0.	0.	.1745E+03	0.
ROW 2	0.	0.	-.4000E+06	0.
ROW 3	0.	0.	0.	0.
ROW 4	0.	0.	0.	0.

--------- MATRIX M3 DIMENSIONED (4X 4)

ROW 1	.1000E+C4	0.	0.	0.
ROW 2	0.	.4000E+05	0.	0.
ROW 3	0.	0.	.1000E+02	0.
ROW 4	0.	0.	0.	.1500E+02

--------- MATRIX FREQM DIMENSIONED (2X 1)

ROW 1	0.
ROW 2	.2000E+01

OUTPUT MATRICES FREQUENCY 1

--------- MATRIX M4 DIMENSIONED (4X 4)

ROW	1	0.	-.4000E+01	-.5797E+02	.4000E+01
ROW	2	0.	.1000E+02	-.4409E+02	.2000E+01
ROW	3	0.	.8000E+01	-.6070E+01	-.2000E+01
ROW	4	0.	.2000E+01	-.1202E+02	.8000E+01

--------- MATRIX M5 DIMENSIONED (4X 4)

ROW	1	.2000E+00	.7000E+01	.7000E+01	.3000E+01
ROW	2	-.5000E+00	.6000E+01	.4000E+01	.2000E+01
ROW	3	-.4000E+00	.1000E+01	.2000E+01	.1000E+01
ROW	4	-.1000E+00	.1000E+01	.1000E+01	.2000E+01

--------- MATRIX C3 DIMENSIONED (8X 1)

ROW	1	.2000E+00
ROW	2	0.
ROW	3	-.5000E+00
ROW	4	0.
ROW	5	-.4000E+00
ROW	6	0.
ROW	7	-.1000E+00
ROW	8	0.

O U T P U T   M A T R I C E S   FREQUENCY   2

--------- MATRIX   M4   DIMENSIONED (4X   4)

ROW	1	0.	-.3000E+01	-.5295E+02	.1000E+01
ROW	2	0.	.6000E+01	-.4110E+02	.1000E+01
ROW	3	0.	.5000E+01	-.1709E+02	-.1000E+01
ROW	4	0.	.2000E+01	-.2103E+02	.3000E+01

--------- MATRIX   M5   DIMENSIONED (4X   4)

ROW	1	.3000E+00	.6000E+01	.6000E+01	.4000E+01
ROW	2	-.6000E+00	.5000E+01	.3000E+01	.3000E+01
ROW	3	-.5000E+00	.2000E+01	.1000E+01	.2000E+01
ROW	4	-.2000E+00	.2000E+01	0.	.2000E+01

--------- MATRIX   C3   DIMENSIONED (8X   1)

ROW	1	.3000E+00
ROW	2	.1000E+00
ROW	3	-.60C0E+00
ROW	4	.2000E+C0
ROW	5	-.5000E+C0
ROW	6	.1000E+C0
ROW	7	-.2000E+00
ROW	8	.3000E+00

L O A D S   M A T R I X   E Q U A T I O N S

TITLE   CHECK CASE 3, MODIFY EOM AND LOADS

O U T P U T   M A T R I C E S   FREQUENCY   1

--------- MATRIX   M1BAR   DIMENSIONED (2X   4)

ROW	1	0.	0.	0.	0.
ROW	2	0.	0.	0.	0.

--------- MATRIX   M2BAR   DIMENSIONED (2X   4)

ROW	1	0.	0.	-.7878E+02	0.
ROW	2	0.	0.	-.2895E+02	0.

--------- MATRIX   M3BAR   DIMENSIONED (2X   4)

ROW	1	.7000E+01	.8000E+01	.4000E+01	.1000E+02
ROW	2	.6000E+01	.3000E+01	-.1000E+01	.5000E+01

```
O U T P U T   M A T R I C E S   FREQUENCY    1

 ---------  MATRIX    M4BAR       DIMENSIONED (     2X      4)

ROW    1     .8000E+01   .4000E+01  -.7349E+01  0.
ROW    2     .6000E+01   .3000E+01   .1552E+02   .2000E+01

 ---------  MATRIX    M5BAR       DIMENSIONED (     2X      4)

ROW    1    -.2000E+01   .1000E+01  -.1000E+01   .5000E+01
ROW    2     .3000E+01  -.1000E+01  -.2000E+01  -.2000E+01

 ---------  MATRIX    C3BAR       DIMENSIONED (     4X      1)

ROW    1    -.2000E+C1
ROW    2    0.
ROW    3     .3000E+01
ROW    4    0.
```

O U T P U T   M A T R I C E S   FREQUENCY 2

```
---------  MATRIX    M4BAR      DIMENSIONED (     2X     4)

ROW    1     .7000E+01    .3000E+01    .1476E+01    .5000E+01
ROW    2     .5000E+01    .2000E+01    .4349E+01    .1000E+01

---------  MATRIX    M5BAR      DIMENSIONED (     2X     4)

ROW    1    -.3000E+01   0.           -.2000E+01    .4000E+01
ROW    2     .2000E+01   0.           -.3000E+01   -.1000E+01

---------  MATRIX    C3BAR      DIMENSIONED (     4X     1)

ROW    1    -.3000E+01
ROW    2     .1000E+00
ROW    3     .2000E+01
ROW    4     .2000E+00
```

```
*******************************************************
*                                                     *
*   PROGRAM L219A2  VERSION JUNE 29,77 IS FINISHED.   *
*   DATE OF RUN IS  77/11/09.                         *
*   TIME OF RUN IS  09.46.25.                         *
*                                                     *
*******************************************************
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*****************************************************
*                                                   *
*   PROGRAM L219A2  VERSION JUNE 29,77 NOW RUNNING. *
*   THE PROGRAM IS PART OF THE DYLOFLX SYSTEM       *
*   DEVELOPED FOR NASA UNDER CONTRACT NAS1-13918.   *
*   DATE OF RUN IS  77/11/09.                       *
*   TIME OF RUN IS  09.46.26.                       *
*                                                   *
*****************************************************
  ($EQMOD    TEST CASE 5                                                 1.0     )
  (TITLE     CHECK CASE 5, MODIFY DERIVATIVES FROM CARDS AND ADD SAS AND SENSORS )
  (SIZE          7          2                                            3.0     )

         P R O B L E M   S I Z E
         NDOF    =     7, TOTAL NUMBER OF DEGREES OF FREEDOM.
         NPAN    =     0, TOTAL NUMBER OF PANELS.
         NFREQM =      2, NUMBER OF FREQUENCIES.
  (OUTPUT    EQEOM          1        EQLOD         1                     4-1     )

         O U T P U T   T A P E S
         IUTEOM = EQEOM      , TAPE NAME FOR MODIFIED EOM MATRICES.
         IFLEOM =           1, FILE POSITION NUMBER OF IUTEOM.
         IUTLOD = EQLOD      , TAPE NAME FOR MODIFIED LOADS MATRICES.
         IFLLOD =          10, FILE POSITION NUMBER OF IUTLOD.
  (PRINT     INPUT     MATRIX    ALL                                     4.2     )

         INPUT MATRICES   P R I N T   OPTION.
         INPR = -999, IF INPR = -999, PRINT ALL INPUT MATRICES.
                              = 0, NO INPUT MATRICES PRINTED.
                              = N, MATRICES OF NTH FREQUENCY ONLY PRINTED.
  (PRINT     OUTPUT    MATRIX    ALL                                     4.3     )

         OUTPUT MATRICES   P R I N T   OPTION.
         IUTPR = -999, IF IUTPR = -999, PRINT ALL OUTPUT MATRICES.
                                = 0, NO OUTPUT MATRICES PRINTED.
                                = N, MATRICES OF NTH FREQUENCY ONLY PRINTED.
                                = 999, PRINT ONLY MODIFIED MATRICES.
  (SYMMETRIC                                                             5.0     )
  ($EOM      EOMTAP         1    1    0    1    1    1    1    0    0    6.0     )

         E Q U A T I O N S   O F   M O T I O N

         INEOM   = EOMTAP     , EOM INPUT TAPE NAME.
         INEOMF  =           1, FILE POSITION NUMBER OF INEOM.

  (DERIVATIVE FROM     CARDS         1     0 2.0      10.0               7.1     )

         DERIVATIVES FOR SYMMETRIC ANALYSIS.
         IVOL    =      CARD, INPUT VOLUME
         NCS     =          1, NUMBER OF CONTROL SURFACE
         INDUN   =          0, UNSTEADY DERIVATIVE INDICATOR
         QUEBAR  =  .200E+01, DYNAMIC PRESSURE
         VT      =  .100E+02, VELOCITY (TRUE AIR SPEED)

         COLUMN NUMBERS OF RIGID BODY FREEDOMS
         IXCOL   =          0, COLUMN OF X FREEDOMS
         IZCOL   =          1, COLUMN OF Z FREEDOMS
         ITCOL   =          2, COLUMN OF THETA FREEDOMS
         COLUMN OF DELTA CONTROL SURFACE FREEDOMS FOLLOW
             4

         CONSTANTS ASSOCIATED WITH DERIVATIVES
         XREF    = 0.        , X-COORDINATE OF MOMENT REFERENCE
```

```
       ZREF    = 0.          , Z-COORDINATE OF MOMENT REFERENCE
       ALPHA1 =  .175E-02, IG ANGLE OF ATTACK
       SW      =  .250E+C1, WING REFERENCE AREA
       CBAR    =  .100E+01, REFERENCE CHORD
       CL1R    =  .3CCE+01, RIGID STEADY STATE DERIVATIVE
       CL1E    =  .20CE+00, ELASTIC STEADY STATE DERIVATIVE

       STEADY STATE DERIVATIVES FROM CARD
       CLU     =  .4C0E+01, C-L-UHAT-RIGID
       CDU     =  .1C0E+01, C-D-UHAT-RIGID
       CMUREF =  .100E+01, C-M-UHAT-RIGID-REF
       CLA     =  .700E+01, C-L-ALPHA-RIGID
       CDA     =  .100E+01, C-D-ALPHA-RIGID
       CMAREF =  .100E+02, C-M-ALPHA-RIGID-REF

       CLUE    =  .100E+01, C-L-UHAT-ELASTIC
       CDUE    =  .100E+00, C-D-UHAT-ELASTIC
       CLAE    = -.100E+01, C-L-ALPHA-ELASTIC
       CDAE    =  .100E+01, C-D-ALPHA-ELASTIC

       CLQ     =  .200E+01, C-L-QUEHAT-RIGID
       CDQ     =  .300E+01, C-D-QUEHAT-RIGID
       CMQREF =  .700E+01, C-M-QUEHAT-RIGID-REF

       CLQE    =  .100E+00, C-L-QHAT-ELASTIC
       CDQE    = -.100E+01, C-D-QHAT-ELASTIC

       CONTROL SURFACE DERIVATIVES
       CLD (C-L-DELTA-RIGID) FOLLOWS
         .400E+01
       CDD (C-D-DELTA-RIGID) FOLLOWS
         .100E+01
       CMDREF (C-M-DELTA-REF) FOLLOWS
         .60CE+01
       CLDE (C-L-DELTA-ELASTIC) FOLLOWS
        -.200E+01
       CDDE (C-D-DELTA-ELASTIC) FOLLOWS
        -.100E+01
(SENSOR                          3    0    1    1                          9.1      )

       INSEN   = LODTP2      , SENSORS INPUT TAPE NAME.
       INSENF =              1, FILE POSITION NUMBER OF INSEN.
       NLDSEN =              3, NUMBER OF LOADS ON SENSOR TAPE.
 MATRIX          IN   OUT
(M2BAR            3     5                                                  9.2-1    )
(M3BAR            1     6                                                  9.2-2    )
(END SENSOR                                                                9.3      )
(SAS                                                                       10.1     )

   ROW   COL M1         M2         M3
(     7     7 1.0                                                          10.2-1   )
(     7     5          -1.0                                                10.2-2   )
(     7     6                      2.0                                     10.2-3   )
(     4     7 1.0       2.0        3.0                                     10.2-4   )
(END SAS                                                                   10.3     )
($QUIT                                                                              )
```

E O M   M A T R I X   E Q U A T I O N S

TITLE   CHECK CASE 5, MODIFY DERIVATIVES FROM CARDS AND ADD SAS AND SENSORS

I N P U T   M A T R I C E S   FREQUENCY   1

--------- MATRIX   M1   DIMENSIONED (4X   4)

ROW	1	0.	0.	0.	0.
ROW	2	0.	0.	0.	0.
ROW	3	0.	0.	.1000E+03	0.
ROW	4	0.	0.	0.	0.

--------- MATRIX   M3   DIMENSIONED (4X   4)

ROW	1	.1000E+C4	0.	0.	0.
ROW	2	0.	.4000E+05	0.	0.
ROW	3	0.	0.	.1000E+02	0.
ROW	4	0.	0.	0.	.1500E+02

S E N S O R

--------- MATRIX   M2BAR   DIMENSIONED (3X   4)

ROW	1	0.	0.	0.	0.
ROW	2	0.	0.	0.	0.
ROW	3	.1CCOE+01	-.1000E+02	.3000E+01	0.

--------- MATRIX   M3BAR   DIMENSIONED (3X   4)

ROW	1	.1000E+01	.1200E+02	.8000E+00	0.
ROW	2	.1000E+01	-.6000E+01	.5000E+00	0.
ROW	3	0.	0.	0.	0.

O U T P U T   M A T R I C E S   FREQUENCY   1

--------- MATRIX   M1   DIMENSIONED (7X   7)

ROW	1	0.	0.	0.	0.	0.	0.	0.
ROW	2	0.	0.	0.	0.	0.	0.	0.
ROW	3	0.	0.	.1000E+03	0.	0.	0.	0.
ROW	4	0.	0.	0.	0.	0.	0.	.1000E+01
ROW	5	0.	0.	0.	0.	-.1000E+01	0.	0.
ROW	6	0.	0.	0.	0.	0.	-.1000E+01	0.
ROW	7	0.	0.	0.	0.	0.	0.	.1000E+01

--------- MATRIX   M2   DIMENSIONED (7X   7)

ROW	1	0.	0.	0.	0.	0.	0.	0.
ROW	2	0.	0.	0.	0.	0.	0.	0.
ROW	3	0.	0.	0.	0.	0.	0.	0.
ROW	4	0.	0.	0.	0.	0.	0.	.2000E+01
ROW	5	.1000E+01	-.1000E+02	.3000E+01	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.	0.
ROW	7	0.	0.	0.	0.	-.1000E+01	0.	0.

--------- MATRIX   M3   DIMENSIONED (7X   7)

ROW	1	.1000E+C4	C.	0.	0.	0.	0.	0.
ROW	2	0.	.4000E+05	0.	0.	0.	0.	0.

ROW	3	0.	0.	.1000E+02	0.	0.	0.	0.
ROW	4	0.	0.	0.	.1500E+02	0.	0.	.3000E+01
ROW	5	0.	0.	0.	0.	0.	0.	0.
ROW	6	.1000E+01	.1200E+02	.8000E+00	0.	0.	0.	0.
ROW	7	0.	0.	0.	0.	0.	.2000E+01	0.

--------- MATRIX FREQM DIMENSIONED (2X 1)

ROW	1	0.
ROW	2	.2000E+01

I N P U T   M A T R I C E S   FREQUENCY   1

--------- MATRIX M4 DIMENSIONED (4X 4)

ROW	1	0.	-.2000E+01	.6000E+01	.2000E+01
ROW	2	0.	.5000E+01	.8000E+01	.1000E+01
ROW	3	0.	.4000E+01	.2000E+01	-.1000E+01
ROW	4	0.	.1000E+01	-.1000E+01	.4000E+01

--------- MATRIX M5 DIMENSIONED (4X 4)

ROW	1	.2000E+00	.7000E+01	.7000E+01	.3000E+01
ROW	2	-.5000E+00	.6000E+01	.4000E+01	.2000E+01
ROW	3	-.4000E+00	.1000E+01	.2000E+01	.1000E+01
ROW	4	-.1000E+00	.1000E+01	.1000E+01	.2000E+01

--------- MATRIX C3 DIMENSIONED (8X 1)

ROW	1	.20CCE+00
ROW	2	0.
ROW	3	-.5000E+00
ROW	4	0.
ROW	5	-.4000E+00
ROW	6	0.
ROW	7	-.10C0E+00
ROW	8	0.

O U T P U T   M A T R I C E S   FREQUENCY   1

--------- MATRIX M4 DIMENSIONED (7X 7)

ROW	1	0.	.3497E+02	.6000E+01	.2001E+02	0.	0.	0.
ROW	2	0.	-.4999E+02	.8000E+01	-.3000E+02	0.	0.	0.
ROW	3	0.	.4000E+01	.2000E+01	-.1000E+01	0.	0.	0.
ROW	4	0.	.10C0E+01	-.1000E+01	.4000E+01	0.	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.	0.
ROW	7	0.	0.	0.	0.	0.	0.	0.

--------- MATRIX M5 DIMENSIONED (7X 7)

ROW	1	.3501E+01	.5013E+00	.7000E+01	.3000E+01	0.	0.	0.
ROW	2	-.5000E+01	-.1750E+01	.4000E+01	.2000E+01	0.	0.	0.
ROW	3	-.4000E+00	.1000E+01	.2000E+01	.1000E+01	0.	0.	0.
ROW	4	-.1000E+00	.1000E+01	.1000E+01	.2000E+01	0.	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.	0.
ROW	7	0.	0.	0.	0.	0.	0.	0.

--------- MATRIX C3 DIMENSIONED (14X 1)

ROW	1	-.3501E+01
ROW	2	0.
ROW	3	.5000E+01
ROW	4	0.
ROW	5	-.4000E+00
ROW	6	0.
ROW	7	-.1000E+00
ROW	8	0.

```
ROW   9    0.
ROW  10    0.
ROW  11    0.
ROW  12    0.
ROW  13    0.
ROW  14    0.
```

I N P U T   M A T R I C E S   FREQUENCY   2

--------- MATRIX M4 DIMENSIONED (4X 4)

ROW	1	0.	-.3000E+01	.7000E+01	.1000E+01
ROW	2	0.	.6000E+01	.9000E+01	.1000E+01
ROW	3	0.	.5000E+01	.3000E+01	-.1000E+01
ROW	4	0.	.2000E+01	-.1000E+01	.3000E+01

--------- MATRIX M5 DIMENSIONED (4X 4)

ROW	1	.30CCE+CO	.6000E+01	.6000E+01	.4000E+01
ROW	2	-.6000E+00	.5000E+01	.3000E+01	.3000E+01
ROW	3	-.5000E+00	.2000E+01	.1000E+01	.2000E+01
ROW	4	-.2000E+00	.2000E+01	0.	.2000E+01

--------- MATRIX C3 DIMENSIONED (8X 1)

ROW	1	.3000E+00
ROW	2	.1000E+00
ROW	3	-.6000E+00
ROW	4	.2000E+00
ROW	5	-.5000E+00
ROW	6	.1000E+00
ROW	7	-.2000E+00
ROW	8	.3000E+00

O U T P U T   M A T R I C E S   FREQUENCY   2

--------- MATRIX M4 DIMENSIONED (7X 7)

ROW	1	0.	.3497E+02	.7000E+01	.2001E+02	0.	0.	0.
ROW	2	0.	-.4999E+02	.9000E+01	-.3000E+02	0.	0.	0.
ROW	3	0.	.5000E+01	.3000E+01	-.1000E+01	0.	0.	0.
ROW	4	0.	.20C0E+01	-.1000E+01	.3000E+01	0.	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.	0.
ROW	7	0.	0.	0.	0.	0.	0.	0.

--------- MATRIX M5 DIMENSIONED (7X 7)

ROW	1	.3501E+01	.5013E+00	.6000E+01	.4000E+01	0.	0.	0.
ROW	2	-.5000E+01	-.1750E+01	.3000E+01	.3000E+01	0.	0.	0.
ROW	3	-.5000E+00	.2000E+01	.1000E+01	.2000E+01	0.	0.	0.
ROW	4	-.2000E+00	.2000E+01	0.	.2000E+01	0.	0.	0.
ROW	5	0.	0.	0.	0.	0.	0.	0.
ROW	6	0.	0.	0.	0.	0.	0.	0.
ROW	7	0.	0.	0.	0.	0.	0.	0.

--------- MATRIX C3 DIMENSIONED (14X 1)

ROW	1	-.3501E+01
ROW	2	.1000E+00
ROW	3	.5000E+01
ROW	4	.2000E+00
ROW	5	-.5000E+00
ROW	6	.1000E+00
ROW	7	-.2000E+00
ROW	8	.3000E+00

ROW	9	0.
ROW	10	0.
ROW	11	0.
ROW	12	0.
ROW	13	0.
ROW	14	0.

```
*********************************************************
*                                                       *
*   PROGRAM L219A2  VERSION JUNE 29,77 IS FINISHED.     *
*   DATE OF RUN IS  77/11/09.                           *
*   TIME OF RUN IS  09.46.28.                           *
*                                                       *
*********************************************************
```

APPENDIX A

RELATIONSHIP BETWEEN INERTIA AND BODY-FIXED AXES FOR A STRAIGHT AND LEVEL REFERENCE CONDITION

This appendix describes the transformation of equations of motion and load equations from inertia to body-fixed axis coordinates. The transformation developed is nonlinear (as shown in appendices B and C) but for straight and level flight, the equations reduce to a linear set. It is this linear transformation that is included in EQMOD.

The differences between inertia and body-fixed axes for small perturbations about a straight and level reference condition is illustrated in figures 13 and 14. In the inertia axes (which are fixed in space), the motion of the body relative to these fixed axes is described by the velocity components in the direction of the fixed axes. Thus the velocities of the body are $\dot{x}'$, $\dot{z}'$ and $\dot{\theta}'$ for symmetric and $\dot{y}'$, $\dot{\phi}'$ and $\dot{\psi}'$ for antisymmetric motions, and the accelerations are $\ddot{x}'$, $\ddot{z}'$, $\ddot{\theta}'$ and $\ddot{y}'$, $\ddot{\phi}'$ and $\ddot{\psi}'$, respectively. In the case of body-fixed axes, the motion is described by the components of the velocity relative to the fixed inertial axes, but in the direction of the moving axes. For this case, the velocities are u, w, and q for symmetric and v, p, and r for antisymmetric motions. Because the axes are rotating, the expressions for acceleration contain products of linear and rotational velocities; they are $\dot{u} + W_1\dot{q}$, $\dot{w} - U_1\dot{q}$, $\dot{q}$, and $(\dot{v} - W_1\dot{p} + U_1\dot{r})$, $\dot{p}$, $\dot{r}$ for symmetric and antisymmetric motions, respectively. The factors U_1 and W_1 are the reference (in this case, 1g) values of velocity in the x' and z' directions. Reference 9 contains a development of the rigid body equation of motion in body-fixed axes.

The symmetric and antisymmetric angles of attack α and β are given by w/U_1 and v/U_1 in body-fixed axes and $z'/U_1 + \theta$ and $y'/U_1 + \alpha_1\phi' - \psi'$ in inertia axes. A point not illustrated in figures 13 and 14, is that the force due to gravity is constant in both magnitude and direction in inertia axes, but in body-fixed axes the weight force develops components along the axes as the axes rotate.

Appendices B and C contain a discussion of the relationship between inertia and body-fixed axes based upon the formulation contained in reference 10. It is shown that for a general reference condition, the symmetric and antisymmetric equations of motion are coupled, and the relationship between inertia and body-fixed axis motions is nonlinear. For the special case of a 1-g level reference condition, the symmetric and antisymmetric equations are uncoupled and the inertia and body-fixed axis motions are related by the following:

$$\begin{aligned} \dot{u} &= \dot{x}' - W_1\,\theta' \\ w &= \dot{z}' + U_1\,\theta' \\ q &= \dot{\theta}' \end{aligned} \tag{A1}$$

$$\begin{aligned} v &= \dot{y}' + W_1\,\phi' - U_1\,\psi' \\ p &= \dot{\phi}' \\ r &= \dot{\psi}' \end{aligned} \tag{A2}$$

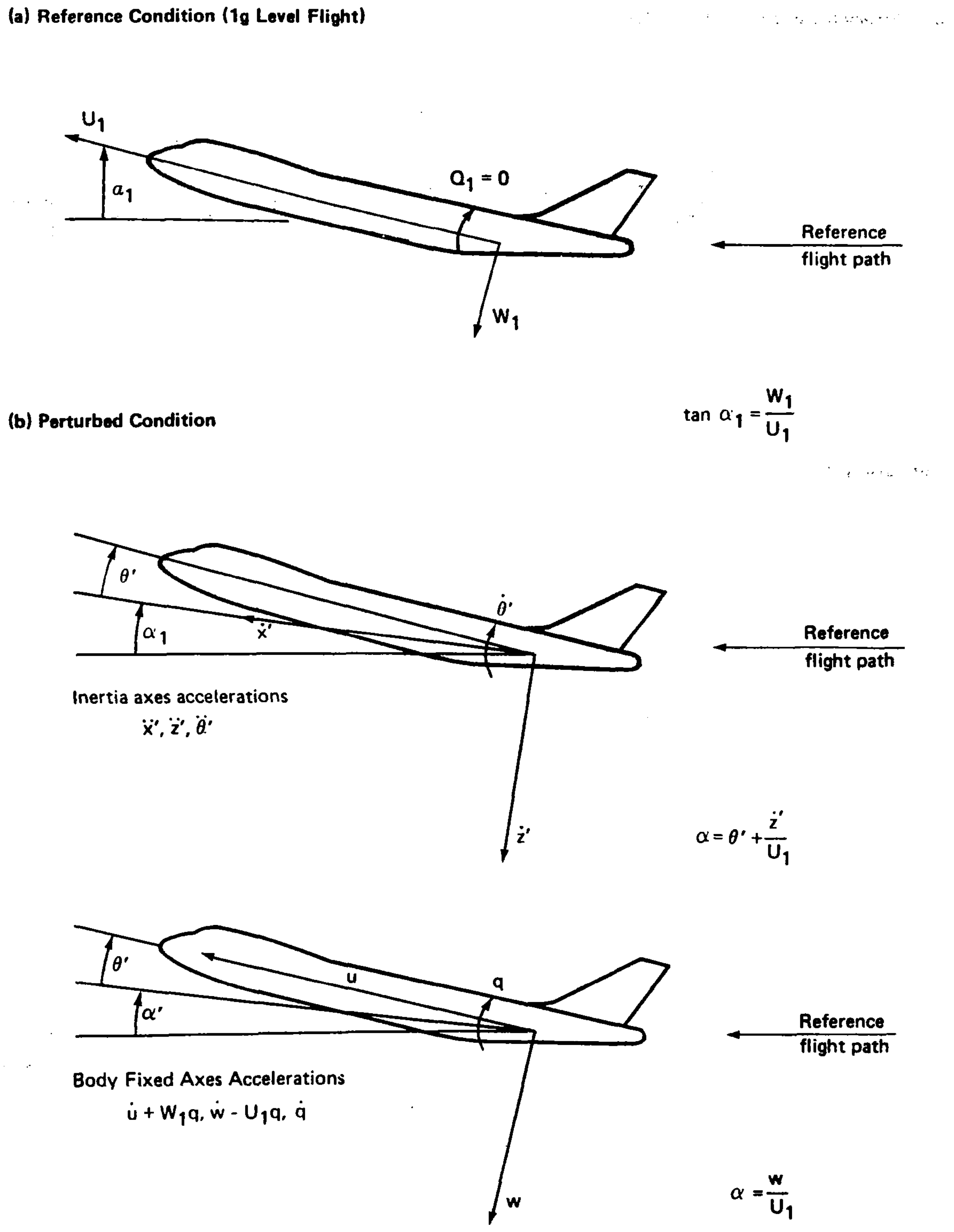


Figure 13. — Inertia and Body-Fixed Axes for Symmetric Perturbations About a 1g Reference Flight Condition

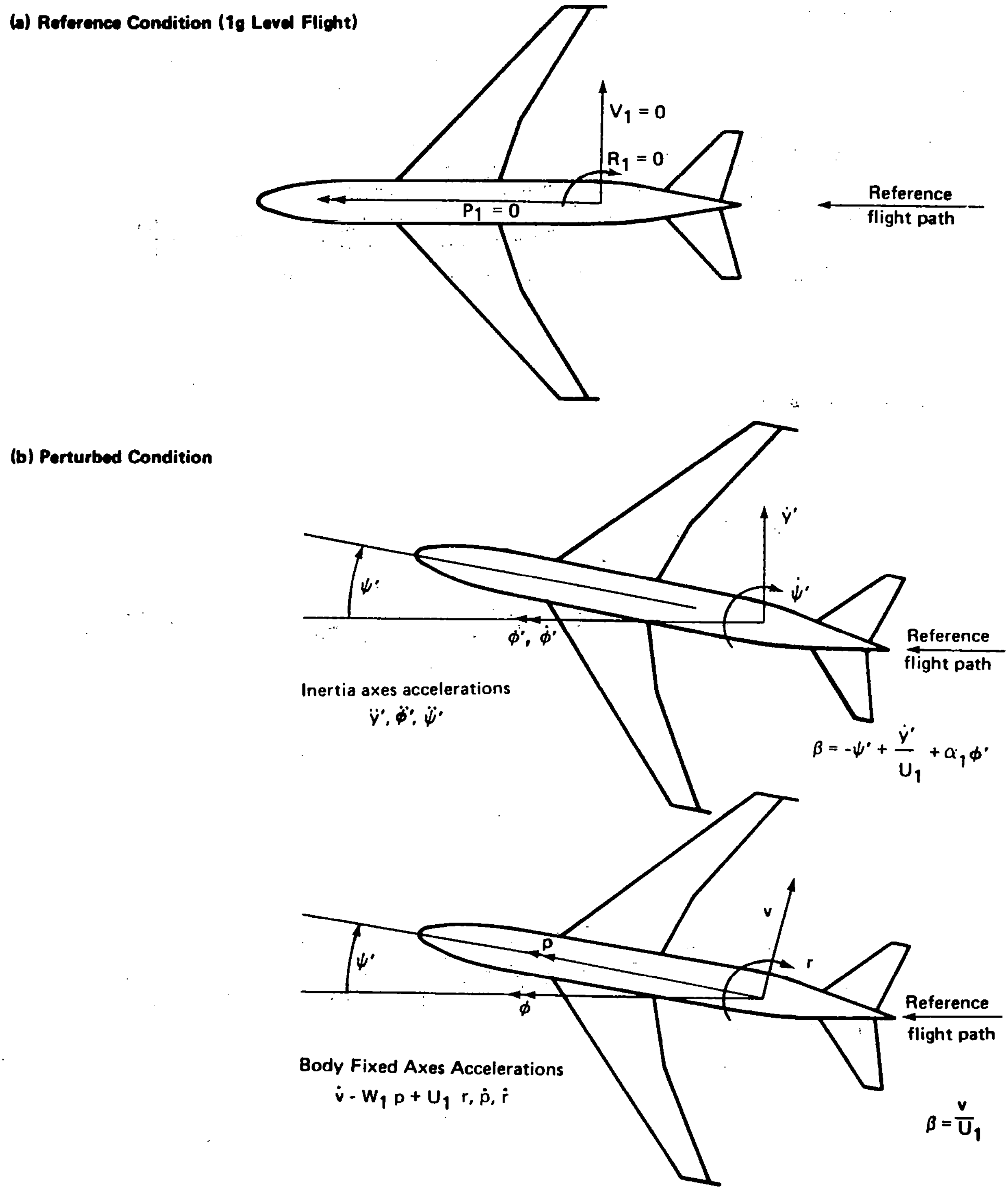


Figure 14. — Inertia and Body-Fixed Axes for Antisymmetric Perturbations About a 1g Reference Flight Condition

EQUATIONS OF MOTION

The rigid body equations of motion for symmetric and antisymmetric motion, assuming α_1 is small, is given in equations (A3) to (A6). The inertia axis equations are simple Newtonian equations, and the body-fixed versions are similar to those derived in reference 9 (eq. (4.14,12) and (4.14,13)). However, equations (A3) to (A6) do not assume that the origin of the axis system is at the c.g., nor that α_1 is zero (stability axes), but does assume a straight and level reference condition (θ_0 in ref. 9 is zero). These equations follow the practice of reference 10 in using a prime for inertia axis quantities.

SYMMETRIC EQUATIONS - Rigid Body Equations of Motion

Inertia Axes

$$M\ddot{x}' + M\Delta z_{cg}\ddot{\theta}' = F_{x'}$$

$$M\ddot{z}' + M\Delta x_{cg}\ddot{\theta}' = F_{z'}$$

$$I_{yy}\ddot{\theta}' + M\Delta z_{cg}\ddot{x}' - M\Delta x_{cg}\ddot{z}' + Mg\Delta z_{cg}\theta' = M_{y'} \tag{A3}$$

Body-Fixed Axes

$$M\dot{u} + M\Delta z_{cg}\dot{q} + MW_1 q + Mg\,\theta' = F_x$$

$$M\dot{w} - M\Delta x_{cg}\dot{q} - MU_1 q = F_z$$

$$I_{yy}\dot{q} + M\Delta z_{cg}\dot{u} - M\Delta x_{cg}\dot{w} + M\Delta z_{cg}W_1 q$$

$$+ M\Delta x_{cg}U_1 q + Mg\Delta z_{cg}\theta' = M_y \tag{A4}$$

ANTISYMMETRIC EQUATIONS

Inertia Axes

$$M\ddot{y}' + M\Delta z_{cg}\ddot{\phi}' + M\Delta x_{cg}\ddot{\psi}' = F_{y'}$$

$$I_{xx}\ddot{\phi}' - I_{xy}\ddot{\psi}' - M\Delta z_{cg}\ddot{y}' + Mg\Delta z_{cg}\phi' = M_{x'}$$

$$I_{zz}\ddot{\psi}' - I_{xz}\ddot{\phi}' + M\Delta x_c\,\ddot{y}' = M_{z'} \tag{A5}$$

Body-Fixed Axes

$$M\dot{v} - M\Delta z_{cg}\dot{p} + M\Delta x_{cg}\dot{r} - MW_1 p + MU_1 r - Mg\,\phi' = F_y$$

$$I_{xx}\dot{p} - I_{xz}\dot{r} - M\Delta z_{cg}\dot{v} + M\Delta z_{cg}W_1 p - M\Delta z_{cg}U_1 r + Mg\Delta z_{cg}\phi' = M_x$$

$$I_{zz}\dot{r} - I_{xz}\dot{p} + M\Delta x_{cg}\dot{v} - M\Delta x_{cg}W_1 p + M\Delta x_{cg}U_1 r - Mg\Delta x_{cg}\phi' = M_z \tag{A6}$$

where:

M = mass

I = inertia

$\Delta x_{c.g.} = x_{REF} - x_{c.g.}$

$\Delta z_{c.g.} = z_{REF} - z_{c.g.}$

AERODYNAMIC FORCES

Aerodynamic forces are normally quoted in terms of lift, drag, side force, and rolling, pitching, and yawing moments (L, D, Y, l, m, n), these quantities being defined along, and normal to, the relative airflow. The aerodynamic forces required in the equations of motion are the components of those forces and moments in the direction of the axes. Since the inertia and body- fixed axes make different angles to the airflow, the expressions for the forces and moments are different. Figure 15 shows these forces and moments for symmetric perturbations assuming that α_1 is small; that is, cos $\alpha_1 = 1$ and sin $\alpha_1 = \alpha_1$. In inertia axes, the angle between the axes and the flight path is $\alpha_1 + \dot{z}'/U_1$, and in body-fixed axes it is $\alpha_1 + w/U_1$; since $w = \dot{z}' + U_1\theta'$, the difference is clearly θ', and the following relationship exists between the inertia axis forces (X', Z', M_y') and the body axis forces (X, Z, M_y):

$$X = X' + L\theta'$$

$$Z = Z - D\theta'$$

$$M_y = M_y' \qquad \text{(A7)}$$

Figure 16 shows the forces for antisymmetric perturbations, and by similar reasoning to the symmetric case:

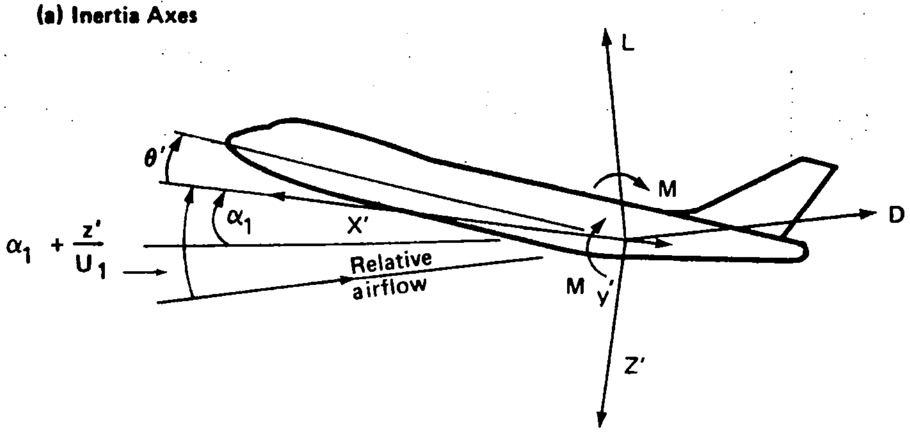
$$Y = Y' - L\phi'$$

$$M_x = M_x'$$

$$M_z = M_z' - m\phi' \qquad \text{(A8)}$$

In the transformation of the roll and yaw moments, the small-angle approximation for α_1 has not been used.

(a) Inertia Axes

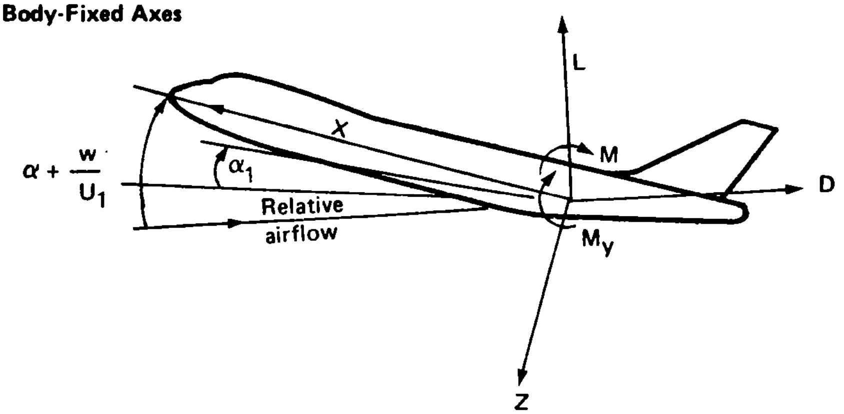


$$X' = L\left(\alpha_1 + \frac{\dot{z}'}{U_1}\right) - D$$

$$Z' = -L - D\left(\alpha_1 + \frac{\dot{z}'}{U_1}\right)$$

$$M_{y'} = m$$

(b) Body-Fixed Axes



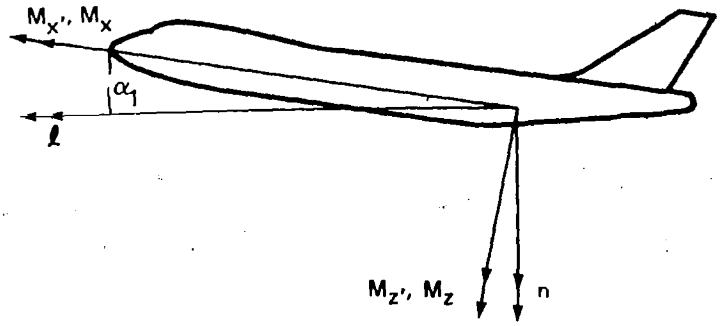
$$X = L\left(\alpha_1 + \frac{w}{U_1}\right) - D$$

$$Z = -L - D\left(\alpha_1 + \frac{w}{U_1}\right)$$

$$M_y = M$$

Figure 15. – Aerodynamic Forces and Moments for Symmetric Perturbations

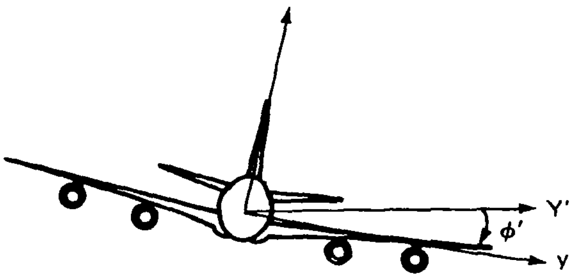
(a) Inertia and Body-fixed axes



$M_{x'} = M_x = \ell \cos \alpha_1 - n \sin \alpha_1$

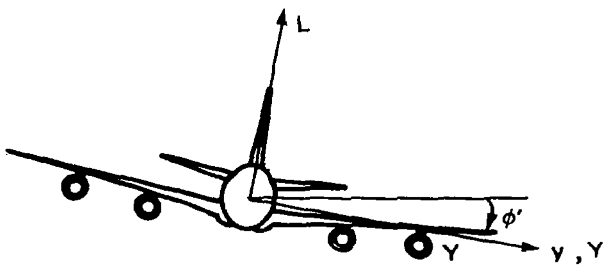
$M_{z'} = M_z + m\phi' = n \cos \alpha_1 + \ell \sin \alpha_1 + m\phi'$

(b) Inertia Axes



$Y' = y + L\phi'$

(c) Body-fixed Axes



$Y = y$

Figure 16. — Aerodynamic Forces and Moments for Antisymmetric Perturbations

AERODYNAMIC DERIVATIVES

Equations (A9) and (A10) show the inertia axis perturbation aerodynamic forces written in terms of the customary aerodynamic derivatives. The nondimensionalizing constants are those adopted in reference 10. The derivation of these expressions from the force equations in figures 15 and 16 is outlined in appendix D.

SYMMETRIC

$$
\begin{aligned}
F_{x'} = \bar{q}\, S_W \Bigg[& \left(C_{L_{\hat{u}}}\alpha_1 - C_{D_{\hat{u}}}\right)\frac{\dot{x}'}{U_1} + \left(C_{L_1} + C_{L_\alpha}\alpha_1 - C_{D_\alpha}\right)\frac{\dot{z}'}{U_1} \\
& + \left(C_{L_{\hat{\dot{\alpha}}}}\alpha_1 - C_{D_{\hat{\dot{\alpha}}}}\right)\frac{\bar{c}}{2U_1}\frac{\ddot{z}'}{U_1} + \left(-C_{L_{\hat{u}}}\alpha_1^2 + C_{L_\alpha}\alpha_1 + C_{D_{\hat{u}}}\alpha_1 - C_{D_\alpha}\right)\theta' \\
& + \left(C_{L_{\hat{q}}}\alpha_1 + C_{L_{\hat{\dot{\alpha}}}}\alpha_1 - C_{D_{\hat{q}}} - C_{D_{\hat{\dot{\alpha}}}}\right)\frac{\bar{c}}{2U_1}\dot{\theta}' + \left(C_{L_{\hat{\dot{q}}}}\alpha_1 - C_{D_{\hat{\dot{q}}}}\right)\frac{\bar{c}^2}{2U_1{}^2}\ddot{\theta}' \\
& + \left(C_{L_\delta}\alpha_1 - C_{D_\delta}\right)\delta + \left(C_{L_{\hat{\dot{\delta}}}}\alpha_1 - C_{D_{\hat{\dot{\delta}}}}\right)\frac{\bar{c}}{2U_1}\dot{\delta}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
F_{z'} = \bar{q}\, S_W \Bigg[& \left(-C_{D_{\hat{u}}}\alpha_1 - C_{L_{\hat{u}}}\right)\frac{\dot{x}'}{U_1} + \left(-C_{D_\alpha}\alpha_1 - C_{L_\alpha}\right)\frac{\dot{z}'}{U_1} \\
& + \left(-C_{D_{\hat{\dot{\alpha}}}}\alpha_1 - C_{L_{\hat{\dot{\alpha}}}}\right)\frac{\bar{c}}{2U_1}\frac{\ddot{z}'}{U_1} + \left(C_{D_{\hat{u}}}\alpha_1{}^2 - C_{D_\alpha}\alpha_1 + C_{L_{\hat{u}}}\alpha_1 - C_{L_\alpha}\right)\theta \\
& + \left(-C_{D_{\hat{\dot{\alpha}}}}\alpha_1 - C_{D_{\hat{q}}}\alpha_1 - C_{L_{\hat{\dot{\alpha}}}} - C_{L_{\hat{q}}}\right)\frac{\bar{c}}{2U_1}\dot{\theta}' + \left(-C_{D_{\hat{\dot{q}}}}\alpha_1 - C_{L_{\hat{\dot{q}}}}\right)\frac{\bar{c}^2}{2U_1{}^2}\ddot{\theta}' \\
& + \left(-C_{D_\delta}\alpha_1 - C_{L_\delta}\right)\delta + \left(C_{D_{\hat{\dot{\delta}}}}\alpha_1 - C_{L_{\hat{\dot{\delta}}}}\right)\frac{\bar{c}}{2U_1}\hat{\dot{\delta}}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
M_{y'} = \bar{q}\, S_W\, \bar{c} \Bigg[& C_{m_{\hat{u}}}\frac{\dot{x}'}{U_1} + C_{m_\alpha}\frac{\dot{z}'}{U_1} + C_{m_{\hat{\dot{\alpha}}}}\frac{\bar{c}}{2U_1}\frac{\ddot{z}'}{U_1} \\
& + \left(-C_{m_{\hat{u}}}\alpha_1 + C_{m_\alpha}\right)\theta' + \left(C_{m_{\hat{\dot{\alpha}}}} + C_{m_{\hat{q}}}\right)\frac{\bar{c}}{2U_1}\dot{\theta}' \\
& + C_{m_{\hat{\dot{q}}}}\frac{\bar{c}^2}{2U_1{}^2}\ddot{\theta}' + C_{m_\delta}\delta + C_{m_{\hat{\dot{\delta}}}}\frac{\bar{c}}{2U_1}\dot{\delta}\Bigg]
\end{aligned}
$$

where:

$\bar{q}$	=	Dynamic pressure
S_W	=	Wing area
$\bar{c}$	=	M.A.C.
b	=	Span
δ	=	Control surface angle

ANTISYMMETRIC

$$F_{y'} = \bar{q}\, S_W \Bigg[C_{y_\beta} \frac{\dot{y}'}{U_1} + C_{y_{\hat{\dot\beta}}} \frac{b}{2U_1} \frac{\ddot{y}'}{U_1} + \left(C_{L_1} + C_{y_\beta} \alpha_1 \right) \phi'$$

$$+ \left(C_{y_{\hat{\dot\beta}}} \alpha_1 + C_{y_{\hat{p}}} \right) \frac{b}{2U_1} \dot{\phi}' + C_{y_{\hat{\dot p}}} \frac{b^2}{4U_1^2} \ddot{\phi}' - C_{y_\beta} \psi'$$

$$+ \left(- C_{y_{\hat{\dot\beta}}} + C_{y_{\hat{r}}} \right) \frac{b}{2U_1} \dot{\psi}' + C_{y_{\hat{\dot r}}} \frac{b^2}{4U_1^2} \ddot{\psi}$$

$$+ C_{y_\delta} \delta + C_{y_{\hat{\dot\delta}}} \frac{b}{2U_1} \dot{\delta} \Bigg]$$

$$M_{x'} = \bar{q}\, S_W\, b \Bigg[\left(C_{\ell_\beta} \cos\alpha_1 - C_{n_\beta} \sin\alpha_1 \right) \frac{\dot{y}'}{U_1} + \left(C_{\ell_{\hat{\dot\beta}}} \cos\alpha_1 - C_{n_{\hat{\dot\beta}}} \sin\alpha_1 \right) \frac{\ddot{y}'}{U_1} + \Big(C_{\ell_\beta} \cos\alpha_1$$

$$- C_{n_\beta} \sin\alpha_1 \Big) \alpha_1 \phi' + \left(C_{\ell_{\hat{\dot\beta}}} \cos\alpha_1\, \alpha_1 - C_{n_{\hat{\dot\beta}}} \sin\alpha_1\, \alpha_1 + C_{\ell_{\hat{p}}} \cos\alpha_1 - C_{n_{\hat{p}}} \sin\alpha_1 \right) \frac{b}{2U_1} \phi'$$

$$+ \left(C_{\ell_{\hat{\dot\rho}}} \cos\alpha_1 - C_{n_{\hat{\dot\rho}}} \sin\alpha_1 \right) \frac{b^2}{4U_1^2} \ddot{\phi}' + \left(- C_{\ell_\beta} \cos\alpha_1 + C_{n_\beta} \sin\alpha_1 \right) \psi'$$

$$+ \left(- C_{\ell_{\hat{\dot\beta}}} \cos\alpha_1 + C_{n_{\hat{\dot\beta}}} \sin\alpha_1 + C_{\ell_{\hat{r}}} \cos\alpha_1 - C_{n_{\hat{r}}} \sin\alpha_1 \right) \frac{b}{2U_1} \dot{\psi}'$$

$$\left(+ C_{\ell_{\hat{\dot r}}} \cos\alpha_1 - C_{n_{\hat{\dot r}}} \sin\alpha_1 \right) \frac{b^2}{4U_1^2} \ddot{\psi}' + \left(C_{\ell_\delta} \cos\alpha_1 - C_{n_\delta} \sin\alpha_1 \right) \delta$$

$$+ \left(C_{\ell_{\hat{\dot\delta}}} \cos\alpha_1 - C_{n_{\hat{\dot\delta}}} \cos\alpha_1 \right) \frac{b}{2U_1} \dot{\delta} \Bigg]$$

$$
\begin{aligned}
M_{z'} = \bar{q}\, S_W\, b \Bigg[& \left(C_{n_\beta} \cos\alpha_1 + C_{\ell_\beta} \sin\alpha_1 \right) \frac{\dot{y}'}{U_1} + \left(C_{n_{\hat{\dot{\beta}}}} \cos\alpha_1 + C_{\ell_{\hat{\dot{\beta}}}} \sin\alpha_1 \right) \frac{\ddot{y}'}{U_1} \\
& + \left(C_{n_\beta} \cos\alpha_1 + C_{\ell_\beta} \sin\alpha_1 \right) \alpha_1\, \phi' + \Big(C_{n_{\hat{\dot{\beta}}}} \cos\alpha_1\, \alpha_1 + C_{\ell_{\hat{\dot{\beta}}}} \sin\alpha_1\, \alpha_1 \\
& + C_{n_{\hat{p}}} \cos\alpha_1 + C_{\ell_{\hat{p}}} \sin\alpha_1 \Big) \frac{b}{2U_1}\, \dot{\phi}' + \left(C_{n_{\hat{\dot{p}}}} \cos\alpha_1 + C_{\ell_{\hat{\dot{p}}}} \sin\alpha_1 \right) \frac{b^2}{4{U_1}^2}\, \ddot{\phi}' \\
& + \left(-C_{n_\beta} \cos\alpha_1 - C_{\ell_\beta} \sin\alpha_1 \right) \psi' + \Big(- C_{n_{\hat{\dot{\beta}}}} \cos\alpha_1 - C_{\ell_{\hat{\dot{\beta}}}} \sin\alpha_1 + C_{n_{\hat{r}}} \cos\alpha_1 \\
& + C_{\ell_{\hat{r}}} \sin\alpha_1 \Big) \frac{b}{2U_1}\, \dot{\psi}' + \left(C_{n_{\hat{\dot{r}}}} \cos\alpha_1 + C_{\ell_{\hat{\dot{r}}}} \sin\alpha_1 \right) \frac{b^2}{4{U_1}^2}\, \ddot{\psi}' \\
& + \left(C_{n_\delta} \cos\alpha_1 + C_{\ell_\delta} \sin\alpha_1 \right) \delta + \left(C_{n_{\hat{\dot{\delta}}}} \cos\alpha_1 + C_{\ell_{\hat{\dot{\delta}}}} \sin\alpha_1 \right) \frac{b}{2U_1}\, \dot{\delta} \Bigg]
\end{aligned}
$$

The DYLOFLEX system is provided with an option in EQMOD to replace the rigid body force elements calculated by the aerodynamic theory with terms calculated from rigid aerodynamic derivatives obtained from FLEXSTAB or any other source. The program will generate these rigid body force elements from the rigid body derivatives. Tables 2 and 3 show these elements and their locations in the equations of motion coefficient matrices. The quantity α_1 in these figures should be understood to be the angle of attack of the coordinate system in 1-g flight. Since α_1 is calculated in an aeroelastic trim analysis, care must be exercised to ensure that α_1 is defined in the same way as in the dynamic analysis. Also, since the value of α_1 in the trim analysis is a function of the assumed structural constraint, an allowance should be made for the difference in constraint between the trim and the dynamic analysis. Tables 2 and 3 assume that the moment derivatives C_ℓ and C_m and C_n are referred to the origin of the DYLOFLEX axis system. This may not be so, and EQMOD will have the facility to transfer these derivatives to the origin.

Table 2. — Formulation of the Rigid-Body Symmetric Generalized Aerodynamic Stiffness and Damping Matrix Elements Using Stability Derivatives

Aerodynamic stiffness matrix $[M_4]$

	^{x}COL	^{z}COL	$^{\theta}COL$	$^{\delta}COL$
^{x}row	0	0	$\bar{q}\,S_W \left(C_{D_{\alpha_R}} + C_{D_{\alpha_E}} - \alpha_1 C_{D_{\hat{u}_R}} - \alpha_1 C_{D_{\hat{u}_E}} - \alpha_1 C_{L_{\alpha_R}} - \alpha_1 C_{L_{\alpha_E}} + \alpha_1^2 C_{L_{\hat{u}_R}} + \alpha_1^2 C_{L_{\hat{u}_E}} \right)$	$\bar{q}\,S_W \left(C_{D_{\delta_R}} + C_{D_{\delta_E}} - \alpha_1 C_{L_{\delta_R}} - \alpha_1 C_{L_{\delta_E}} \right)$
^{z}row	0	0	$\bar{q}\,S_W \left(C_{L_{\alpha_R}} - \alpha_1 C_{L_{u_R}} + \alpha_1 C_{D_{\alpha_R}} - \alpha_1^2 C_{D_{\hat{u}_R}} \right)$	$\bar{q}\,S_W \left(C_{L_{\delta_R}} + \alpha_1 C_{D_{\delta_R}} \right)$
$^{\theta}row$	0	0	$\bar{q}\,S_W\,\bar{c} \left(-C_{m_{\alpha_R}} + \alpha_1 C_{m_{\hat{u}_R}} \right)$	$-\bar{q}\,S_W\,\bar{c}\,C_{m_{\delta_R}}$

Aerodynamic damping matrix $[M_5]$

	^{x}COL	^{z}COL	$^{\theta}COL$	$^{\delta}COL$
^{x}row	$\frac{\bar{q}\,S_W}{U_1} \left(C_{D_{\hat{u}_R}} + C_{D_{\hat{u}_E}} - \alpha_1 C_{L_{\hat{u}_R}} - \alpha_1 C_{L_{\hat{u}_E}} \right)$	$\frac{\bar{q}\,S_W}{U_1} \left(C_{D_{\alpha_R}} + C_{D_{\alpha_E}} - \alpha_1 C_{L_{\alpha_R}} - \alpha_1 C_{L_{\alpha_E}} - C_{L_{1_R}} - C_{L_{1_E}} \right)$	$\frac{\bar{q}\,S_W\,\bar{c}}{2\,U_1} \left(C_{D_{\hat{q}_R}} + C_{D_{\hat{q}_E}} + C_{D_{\hat{\dot{\alpha}}_E}} + C_{D_{\hat{\dot{\alpha}}_E}} - \alpha_1 C_{L_{\hat{q}_R}} - \alpha_1 C_{L_{\hat{q}_E}} - \alpha_1 C_{L_{\hat{\dot{\alpha}}_R}} - \alpha_1 C_{L_{\hat{\dot{\alpha}}_E}} \right)$	0
^{z}row	$\frac{\bar{q}\,S_W}{U_1} \left(C_{L_{\hat{u}_R}} + \alpha_1 C_{D_{\hat{u}_R}} \right)$	$\frac{\bar{q}\,S_W}{U_1} \left(C_{L_{\alpha_R}} + \alpha_1 C_{D_{\alpha_R}} \right)$	$\frac{\bar{q}\,S_W\,\bar{c}}{2\,U_1} \left(C_{L_{\hat{q}_R}} + C_{L_{\hat{\dot{\alpha}}_R}} + \alpha_1 C_{D_{\hat{q}_R}} + \alpha_1 C_{D_{\hat{\dot{\alpha}}_R}} \right)$	0
$^{\theta}row$	$-\frac{\bar{q}\,S_W\,\bar{c}}{U_1} C_{m_{\hat{u}_R}}$	$-\frac{\bar{q}\,S_W\,\bar{c}}{U_1} C_{m_{\alpha_R}}$	$\frac{\bar{q}\,S_W\,\bar{c}^2}{2\,U_1} \left(-C_{m_{\hat{q}_R}} - C_{m_{\hat{\dot{\alpha}}_R}} \right)$	0

^{x}COL, ^{z}COL, $^{\theta}COL$, and $^{\delta}COL$ are the column locations of the x, z, θ, and δ freedoms.
(Note: There may be more than one control surface freedom.)

The $[M_4]$, $[M_5]$ elements are defined in the inertial axis system.

Table 3. —Formulation of the Rigid-Body Antisymmetric Generalized Aerodynamic Stiffness and Damping Matrix Elements Using Stability Derivatives

Aerodynamic stiffness matrix $[M_4]$

	y_{COL}	ϕ_{COL}	ψ_{COL}	δ_{COL}
y_{ROW}	0	$\bar{q}\, S_W(-C_{L_{1_R}} - C_{L_{1_E}} - \alpha_1 C_{y_{\beta_R}})$	$\bar{q}\, S_W C_{y_{\beta_R}}$	$-\bar{q}\, S_W C_{y_{\delta_R}}$
ψ_{ROW}	0	$\bar{q}\, S_W b(-C_{\ell_{\beta_R}} \cos\alpha_1 + C_{n_{\beta_R}} \sin\alpha_1)\alpha_1$	$\bar{q}\, S_W b(C_{\ell_{\beta_R}} \cos\alpha_1 - C_{n_{\beta_R}} \sin\alpha_1)$	$\bar{q}\, S_W b(-C_{\ell_{\delta_R}} \cos\alpha_1 + C_{n_{\delta_R}} \sin\alpha_1)$
ϕ_{ROW}	0	$\bar{q}\, S_W b(-C_{n_{\beta_R}} \cos\alpha_1 - C_{\ell_{\beta_R}} \sin\alpha_1)\alpha_1$	$\bar{q}\, S_W b(C_{n_{\beta_R}} \cos\alpha_1 + C_{\ell_{\beta_R}} \sin\alpha_1)$	$\bar{q}\, S_W b(-C_{n_{\delta_R}} \cos\alpha_1 - C_{\ell_{\delta_R}} \sin\alpha_1)$

Aerodynamic damping matrix $[M_5]$

	y_{COL}	ϕ_{COL}	ψ_{COL}	δ_{COL}
y_{ROW}	$-\dfrac{\bar{q}\, S_W}{U_1} C_{y_{\beta_R}}$	$\dfrac{\bar{q}\, S_W b}{2U_1}\left(-C_{y_{\hat{p}_R}} - \alpha_1 C_{y_{\hat{\dot{\beta}}_R}}\right)$	$\dfrac{\bar{q}\, S_W b}{2U_1}\left(-C_{y_{\hat{r}_R}} + C_{y_{\hat{\dot{\beta}}_R}}\right)$	0
ϕ_{ROW}	$\dfrac{\bar{q}\, S_W b}{U_1}\left(-C_{\ell_{\beta_R}} \cos\alpha_1 + C_{n_{\beta_R}} \sin\alpha_1\right)$	$\dfrac{\bar{q}\, S_W b^2}{2U_1}\left(-C_{\ell_{\hat{p}_R}} \cos\alpha_1 + C_{n_{\hat{p}_R}} \sin\alpha_1 - C_{\ell_{\hat{\dot{\beta}}_R}} (\cos\alpha_1)\, \alpha_1 + C_{n_{\hat{\dot{\beta}}_R}} (\sin\alpha_1)\, \alpha_1\right)$	$\dfrac{\bar{q}\, S_W b^2}{2U_1}\left(-C_{\ell_{\hat{r}_R}} \cos\alpha_1 + C_{n_{\hat{r}_R}} \sin\alpha_1 + C_{\ell_{\hat{\dot{\beta}}_R}} \cos\alpha_1 - C_{n_{\hat{\dot{\beta}}_R}} \sin\alpha_1\right)$	0
ψ_{ROW}	$\dfrac{\bar{q}\, S_W b}{U_1}\left(-C_{n_{\beta_R}} \cos\alpha_1 - C_{\ell_{\beta_R}} \sin\alpha_1\right)$	$\dfrac{\bar{q}\, S_W b^2}{2U_1}\left(-C_{n_{\hat{p}_R}} \cos\alpha_1 - C_{\ell_{\hat{p}_R}} \sin\alpha_1 - C_{n_{\hat{\dot{\beta}}_R}} (\cos\alpha_1)\alpha_1 - C_{\ell_{\hat{\dot{\beta}}_R}} (\sin\alpha_1)\alpha_1\right)$	$\dfrac{\bar{q}\, S_W b^2}{2U_1}\left(-C_{n_{\hat{r}_R}} \cos\alpha_1 - C_{\ell_{\hat{r}_R}} \sin\alpha_1 + C_{n_{\hat{\dot{\beta}}_R}} \cos\alpha_1 + C_{\ell_{\hat{\dot{\beta}}_R}} \sin\alpha_1\right)$	0

The $[M_4]$ and $[M_5]$ elements are defined in the inertial axis system

TRANSFORMATION TO BODY-FIXED COORDINATES

DYLOFLEX develops the dynamic equations of motion and load equations using inertia axes and provides an option to transform the coordinates to body axis coordinates by means of equations (A1) and (A2). The following equations show the application of this transformation to equations (A3) to (A6) for a straight and level reference condition. The equations that result are the same as the body-fixed equations except that the weight term in the x and y equations appears as a lift term.

The method of implementing the transformation in EQMOD is shown in section 4.5.

From Equations (A1):

$$\ddot{x}' = u + W_1 \dot{\theta}' = \dot{u} + W_1 q$$

$$\ddot{z}' = \dot{w} - U_1 \dot{\theta}' = \dot{w} - U_1 q$$

$$\ddot{\theta}' = \dot{q}$$

Substituting in Equations (A3)

$$M\dot{u} + M\Delta z_{cg} \dot{q} + MW_1 q = F_{x'}$$

$$M\dot{u} - M\Delta x_{cg} \dot{q} - MU_1 q = F_{z'}$$

$$I_{yy} \dot{q} + M\Delta z_{cg} \dot{u} - M\Delta x_{cg} \dot{w} + M\Delta z_{cg} W_1 q + M\Delta x_{cg} U_1 q + Mg\Delta z_{cg} \theta' = M_y'$$

From Equations (A7)

$$F_{x'} = F_x - \bar{q} S_W C_{L_1} \theta'$$

$$F_{z'} = F_z$$

$$M_{y'} = M_y$$

Therefore:

$$M\dot{u} + M\Delta z_{cg.} \dot{q} + MW_1 q = F_x - q S_W C_{L_1} \theta'$$

$$M\dot{w} - M\Delta x_{cg} \dot{q} - MU_1 q = F_z$$

$$I_{yy} \dot{q} + M\Delta z_{cg} \dot{u} - M\Delta x_{cg} \dot{w} + M\Delta z_{cg} W_1 \bar{q} + M\Delta x_{cg} U_1 q$$

$$+ Mg\Delta z_{cg} \theta' = M_y$$

Since for a 1g Reference

$$\bar{q} S_W C_{L_1} = Mg$$

The Above Equations are the Same as Equations (A4)

From Equations (A2)

$$\ddot{y}' = \dot{v} - W_1 \dot{\phi}' + U_1 \dot{\psi}' = \dot{v} - W_1 p + U_1 r$$

$$\ddot{\phi}' = \dot{p}$$

$$\ddot{\psi}' = \dot{r}$$

Substituting in Equations (A5)

$$M\dot{v} - M\Delta z_{cg} \dot{p} + M\Delta x_{cg} \dot{r} - MW_1 p + MU_1 r = F_{y'}$$

$$I_{xx} \dot{p} - I_{xz} \dot{r} - M\Delta z_{cg} \dot{v} + M\Delta z_{cg} W_1 p - M\Delta z_{cg} U_1 r + Mg\Delta z_{cg} \phi' = M_{x'}$$

$$I_{zz} \dot{r} - I_{xz} \dot{p} + M\Delta x_{cg} \dot{v} - M\Delta x_{cg} W_1 p + M\Delta x_{cg} U_1 r = M_{z'}$$

From Equations (A8)

$$F_{y'} = F_y + \bar{q} S_W C_{L_1} \phi'$$

$$M_{x'} = M_x$$

$$M_{z'} = M_z + \bar{q} S_W \bar{c} C_{m_1} \phi'$$

Therefore:

$$M\dot{v} - M\Delta z_{cg} \dot{p} + M\Delta x_{cg} \dot{r} - MW_1 p + MU_1 r = F_y + \bar{q} S_W C_{L_1} \phi'$$

$$I_{xx} \dot{p} - I_{xz} \dot{r} - M\Delta z_{cg} \dot{v} + M\Delta z_{cg} W_1 p - M\Delta z_{cg} U_1 r + Mg\Delta z_{cg} \phi' = M_x$$

$$I_{zz} \dot{r} - I_{xz} \dot{p} + M\Delta x_{cg} \dot{v} - M\Delta x_{cg} W_1 p + M\Delta x_{cg} U_1 r = M_z + \bar{q} S_W \bar{c} C_{m_1} \phi'$$

Since:

$$\bar{q} S_W C_{L_1} = Mg$$

and

$$\bar{q} S_W \bar{c} C_{m_1} = Mg\Delta x_{cg}$$

The Above are the Same as Equations (A6)

DISCUSSION

It is shown that the simple transformation from inertia to body-fixed axis coordinates is a valid operation that should render the data more acceptable to groups accustomed to body-fixed axes and assist in comparing rigid body derivatives. This procedure, however, is valid only if the reference condition is straight and level flight.

It should be pointed out that DYLOFLEX defines motions in a set of inertia axes with $\mathbf{x}$ defined in some convenient direction along the body. These axes are the inertia equivalent of what reference 9 (which uses body-fixed axes) refers to as "body" axes. Equally well, however, $\mathbf{x}$ could be defined along the reference flight path; in which case, the axes would be the inertia equivalent of "stability" axes. If these axes are used, then $\alpha_1 = 0$. Provided α_1 is reasonally small, it can be (and usually is) neglected in symmetric dynamic analyses. However, because of the effect it has on roll/yaw coupling which, in turn, affects the dutch roll response, it should not be neglected in antisymmetric cases.

APPENDIX B

RELATIONSHIP BETWEEN INERTIA AND BODY-FIXED AXES EQUATIONS OF MOTION

Figure 17 shows the Laplace transform of the linearized general perturbation equations for an elastic airplane. The coordinates are:

u_p, v_p, w_p, p_p, q_p, r_p	Perturbation linear and angular velocities relative to a set of body-fixed axes with the origin at the airplane c.g.
x_p', y_p', z_p'	Perturbation displacements of the c.g. relative to a set of inertial axes. These axes are oriented to the horizontal through the constant Euler angles θ_0, ϕ_0, ψ_0.
$\theta_p \phi_p, \psi_p$	Perturbation values of the Euler angles (rotations of the body-fixed axes relative to the inertia axes).

The subscript 1 is used to denote the reference values of the above.

U_{eS}, U_{eA}	Perturbation values of symmetric and antisymmetric elastic coordinates chosen so that their contribution to the total linear and angular momentum of the airplane is zero. In other words, these coordinates are associated with free-free normal modes of vibration of the structure.

In these equations:

M	Airplane mass.
$I_{xx}, I_{yy}, I_{zz}, I_{xz}$	Airplane inertias.
S	Laplace operator.
M_S, D_S, M_A, D_A, K_A	The generalized structural mass, damping, and stiffness matrices for the symmetric and antisymmetric elastic coordinates.
$F_x, F_y, F_z, M_x, M_y, M_z$ Q_S, Q_A	External forces and moments.
A_{ij}	Time variant coefficients (functions of θ_1, ϕ_1, ψ_1).

The equations assume that the airplane is symmetrical. The coordinates have been arranged into symmetric and antisymmetric groups. In each group, the first three equations are the rigid body Euler equations of motion, the next three are the auxiliary equations, and the last are the elastic equations. Since the inertia coordinates x_p', y_p', and z_p' do not appear in the equations of motion, it is usual to omit these equations. There is no coupling between the elastic coordinates and the rigid body coordinates since the elastic modes are free-free. The elastic deformations contribute to the external forces and moments however.

The coefficients A_{ij} are shown in appendix C. These coefficients are functions of θ_1, ϕ_1, and ψ_1 (the reference values of the Euler angles) and since θ_1, ϕ_1, and ψ_1 are time variant, the coefficients are time variant. It should be noted that for a special set of inertia axes where z′ is aligned with the gravity vector, $\theta_0 = \phi_0 = 0$ and the equations become identical to those shown in reference 10 (equations (6.2.29), (6.5.16) and (6.5.17)). The only time variant terms in the equations of motion are those associated with the gravity vector and it is usual to solve the equations of motion assuming that they are approximately constant. As can be seen from appendix C, the condition for these terms to be constant (if θ_0 and ϕ_0 are zero) is: $\theta_1 = \phi_1 = 0$.

Even if the gravity terms are assumed constant, the transformation from body-fixed to inertia coordinates (given by the auxiliary equations) is in general nonlinear, since the coefficients are functions of θ_1, ϕ_1, and ψ_1 . All the coefficients in figure 17 become time invariant if $\theta_1 = \phi_1 = \psi_1 = 0$. As shown in appendix C, this implies that $P_1 = Q_1 = R_1 = 0$ (that is, the reference flight condition is rectilinear. Since θ_1, ϕ_2, and ψ_1 are constant, the inertia axes may be oriented so that $\theta_1 = \phi_1 = \psi = 0$. Figure 18 shows the equations for this case.

Applying the transformation given by the auxiliary equations to the equations of motion results in the Lagrangian form of the equations shown in figure 19.

If the reference condition is straight and level flight, then $\phi_0 = V_1 = 0$ and tan $\theta_0 = W_1/U_1$. If θ_0 is put to zero, the axes become inertia stability axes and the equations are shown in figure 20. The equations of motion in figure 20 are those generally used for loads and the symmetric and antisymmetric analyses are carried out independently. The forces and moments on the right-hand side are, of course, written in terms of the inertia coordinates. The auxiliary equations are used primarily to transform aerodynamic wind tunnel data, which is normally quoted in terms of body-fixed axis coordinates, into inertia coordinates. For example, figure 20 shows perturbation α and β in both body-fixed and inertia coordinates.

$$
\begin{bmatrix}
MS & MQ_1 & MW_1 & & & A_{u,\theta} & & -MR_1 & & -MV_1 & & & A_{u,\psi} & \\
-MQ_1 & MS & -MU_1 & & & A_{w,\theta} & & MP_1 & MV_1 & & & A_{w,\phi} & A_{w,\psi} & \\
 & & I_{yy}S & & & & & & \begin{matrix}2P_1 I_{xz} \\ + \\ R_1 (I_{xx}-I_{zz})\end{matrix} & \begin{matrix}-2R_1 I_{xz} \\ + \\ P_1 (I_{xx}-I_{zz})\end{matrix} & & & & \\
A_{x',u} & A_{x',w} & & -S & & A_{x',\theta} & & A_{x',v} & & & & A_{x',\phi} & A_{x',\psi} & \\
A_{z',u} & A_{z',w} & & & -S & A_{z',\theta} & & A_{z',v} & & & & A_{z',\phi} & & \\
 & & -\cos\phi_1 & & & S & & & & \sin\phi_1 & & \dot{\Psi}_1 \cos\theta_1 & & \\
 & & & & & & \begin{matrix}\tilde{m}_S S^2 \\ + \tilde{D}_S S \\ + \tilde{K}_S\end{matrix} & & & & & & & \\
MR_1 & -MP_1 & & & & A_{v,\theta} & & MS & & & & A_{v,\phi} & A_{v,\psi} & \\
 & & \begin{matrix}-P_1 I_{xz} \\ + \\ R_1 (I_{zz}-I_{yy})\end{matrix} & & & & & & \begin{matrix}I_{xx} S \\ - Q_1 I_{xz}\end{matrix} & \begin{matrix}-I_{xz} S \\ + Q_1 (I_{zz}-I_{yy})\end{matrix} & & & & \\
 & & \begin{matrix}R_1 I_{xz} \\ + \\ P_1 (I_{yy}-I_{xx})\end{matrix} & & & & & & \begin{matrix}-I_{xz} S \\ + \\ Q_1 (I_{yy}-I_{xx})\end{matrix} & \begin{matrix}I_{zz} S \\ + \\ Q_1 I_{xz}\end{matrix} & & & & \\
A_{y',u} & A_{y',w} & & & & A_{y',\theta} & & A_{y',v} & & & -S & A_{y',\phi} & A_{y',\psi} & \\
 & & \begin{matrix}-\sin\phi_1 \\ \tan\theta_1\end{matrix} & & & -\dot{\Psi}_1 \sec\theta_1 & & & -1 & \begin{matrix}-\cos\phi_1 \\ \tan\theta_1\end{matrix} & & \begin{matrix}S \\ -\dot{\theta}_1 \tan\theta_1\end{matrix} & & \\
 & & \begin{matrix}-\sin\phi_1 \\ \sec\theta_1\end{matrix} & & & -\dot{\Psi}_1 \tan\theta_1 & & & & \begin{matrix}-\cos\phi_1 \\ \sec\theta_1\end{matrix} & & -\dot{\theta}_1 \sec\theta_1 & S & \\
 & & & & & & & & & & & & & \begin{matrix}\bar{M}_A S^2 \\ + \bar{D}_A S \\ + \bar{K}_A\end{matrix}
\end{bmatrix}
\begin{bmatrix}
\bar{u}_p \\ \bar{w}_p \\ \bar{q}_p \\ \bar{x}'_p \\ \bar{z}'_p \\ \bar{\theta}_p \\ \bar{\eta}_{e_S} \\ \bar{v}_p \\ \bar{p}_p \\ \bar{r}_p \\ \bar{y}'_p \\ \bar{\phi}_p \\ \bar{\psi}_p \\ \bar{\eta}_{e_A}
\end{bmatrix}
=
\begin{bmatrix}
\bar{F}_x \\ \bar{F}_z \\ \bar{M}_y \\ 0 \\ 0 \\ 0 \\ \bar{Q}_S \\ \bar{F}_y \\ \bar{M}_x \\ \bar{M}_z \\ 0 \\ 0 \\ 0 \\ \bar{Q}_A
\end{bmatrix}
\begin{array}{l}
\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{SYMMETRIC EQUATIONS OF MOTION} \\
\left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{AUXILIARY EQUATIONS} \\
\left.\begin{matrix} \\ \end{matrix}\right\} \text{EULER EQUATION} \\
\left.\begin{matrix} \\ \end{matrix}\right\} \text{SYMMETRIC STRUCTURAL EQUATIONS} \\
\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{ANTISYMMETRIC EQUATIONS OF MOTION} \\
\left.\begin{matrix} \\ \end{matrix}\right\} \text{AUXILIARY EQUATION} \\
\left.\begin{matrix} \\ \\ \end{matrix}\right\} \text{EULER EQUATIONS} \\
\left.\begin{matrix} \\ \end{matrix}\right\} \text{ANTISYMMETRIC STRUCTURAL EQUATIONS}
\end{array}
$$

Figure 17. —General Perturbation Equations

$$
\left[\begin{array}{cccccccccccccc}
MS & & MW_1 & & & Mg\cos\phi_o\cos\theta_o & & & & -MV_1 & & & -Mg\sin\phi_o\cos\theta_o & \\
 & MS & -MU_1 & & & -Mg\sin\theta_o & & & MV_1 & & & Mg\sin\phi_o\cos\theta_o & & \\
 & & I_{yy}S & & & & & & & & & & & \\
1 & & & -S & & W_1 & & & & & & & -V_1 & \\
 & 1 & & & -S & -U_1 & & & & & & V_1 & & \\
 & & -1 & & & S & & & & & & & & \\
 & & & & & & \bar{m}_s S^2 + \bar{D}_s S + \bar{K}_s & & & & & & & \\
 & & & & & & & MS & -MW_1 & MU_1 & & -Mg\cos\phi_o\cos\theta_o & Mg\sin\theta_o & \\
 & & & & & & & & I_{xx}S & -I_{xz}S & & & & \\
 & & & & & & & & -I_{xz}S & I_{zz}S & & & & \\
 & & & & & & & 1 & & & -S & -W_1 & U_1 & \\
 & & & & & & & & -1 & & & S & & \\
 & & & & & & & & & -1 & & & S & \\
 & & & & & & & & & & & & & \bar{M}_A S^2 + \bar{D}_A S + \bar{K}_A
\end{array}\right]
\left[\begin{array}{c}
\bar{u}_p \\ \bar{w}_p \\ \bar{q}_p \\ \bar{x}'_p \\ \bar{z}'_p \\ \bar{\theta}'_p \\ \bar{u}_{e_s} \\ \bar{v}_p \\ \bar{p}_p \\ \bar{r}_p \\ \bar{y}'_p \\ \bar{\phi}'_p \\ \bar{\psi}'_p \\ \bar{u}_{e_A}
\end{array}\right]
=
\left[\begin{array}{c}
\bar{F}_x \\ \bar{F}_z \\ \bar{M}_y \\ 0 \\ 0 \\ 0 \\ \bar{Q}_s \\ \bar{F}_y \\ \bar{M}_x \\ \bar{M}_z \\ 0 \\ 0 \\ 0 \\ \bar{Q}_A
\end{array}\right]
$$

Note The inertia axes are oriented to coincide with the body axes for the reference flight condition ($\theta_1 = \phi_1 = \psi_1 = 0$)

Figure 18. — Perturbation Equations for Rectilinear Reference Flight Conditions

$$
\begin{bmatrix}
1 & & & -S & & W_1 & & & & & & & -V_1 & \\
& 1 & & & -S & -U_1 & & & & & & V_1 & & \\
& & -1 & & & S & & & & & & & & \\
& & & MS^2 & & \begin{matrix} Mg \\ \cos\phi_o \\ \cos\theta_o \end{matrix} & & & & & & & \begin{matrix} -Mg \\ \sin\phi_o \\ \cos\theta_o \end{matrix} & \\
& & & & MS^2 & \begin{matrix} Mg \\ \sin\theta_o \end{matrix} & & & & & & \begin{matrix} Mg \\ \sin\phi_o \\ \cos\theta_o \end{matrix} & & \\
& & & & & I_{yy}S^2 & & & & & & & & \\
& & & & & & \begin{matrix} \bar{m}_s S^2 \\ +\bar{D}_s S \\ +\bar{K}_s \end{matrix} & & & & & & & \\
& & & & & & & 1 & & & -S & -W_1 & U_1 & \\
& & & & & & & & -1 & & & S & & \\
& & & & & & & & & -1 & & & S & \\
& & & & & & & & & & MS^2 & \begin{matrix} -Mg \\ \cos\phi_o \\ \cos\theta_o \end{matrix} & \begin{matrix} Mg \\ \sin\theta_o \end{matrix} & \\
& & & & & & & & & & & I_{xx}S & -I_{xz}S & \\
& & & & & & & & & & & -I_{xz}S & I_{zz}S & \\
& & & & & & & & & & & & & \begin{matrix} \bar{m}_A S^2 \\ +\bar{D}_A S \\ +\bar{K}_A \end{matrix}
\end{bmatrix}
\begin{bmatrix}
\bar{u}_p \\ \bar{w}_p \\ \bar{q}_p \\ \bar{x}'_p \\ \bar{z}'_p \\ \bar{\theta}'_p \\ \bar{u}_{e_s} \\ \bar{v}_p \\ \bar{p}_p \\ \bar{r}_p \\ \bar{y}'_p \\ \bar{\phi}'_p \\ \bar{\psi}'_p \\ \bar{u}_{e_A}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ \bar{F}_{x_p} \\ \bar{F}_{z_p} \\ \bar{M}_{y_p} \\ \bar{Q}_s \\ 0 \\ 0 \\ 0 \\ \bar{F}_{y_p} \\ \bar{M}_{x_p} \\ \bar{M}_{z_p} \\ \bar{Q}_A
\end{bmatrix}
\begin{matrix}
\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{AUXILIARY EQUATIONS} \\
\left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{SYMMETRIC EQUATIONS OF MOTION} \\
\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{AUXILIARY EQUATIONS} \\
\left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} \text{ANTISYMMETRIC EQUATIONS OF MOTION}
\end{matrix}
$$

Figure 19. — Lagrangian Perturbation Equations for a Rectilinear Reference Condition

$$
\left[\begin{array}{cccccccccccccc}
1 & & & -S & & & & & & & & & & \\
& 1 & & & -S & -U_1 & & & & & & & & \\
& & -1 & & & S & & & & & & & & \\
& & & MS^2 & & M_g & & & & & & & & \\
& & & & MS^2 & & & & & & & & & \\
& & & & & I_{yy}S^2 & & & & & & & & \\
& & & & & & \begin{array}{c}\overline{m}_s S^2\\ +\overline{D}_s S\\ +\overline{K}_s\end{array} & & & & & & & \\
\hline
& & & & & & & 1 & & & -S & & U_1 & \\
& & & & & & & & -1 & & & S & & \\
& & & & & & & & & -1 & & & S & \\
& & & & & & & & & & MS^2 & -Mg & & \\
& & & & & & & & & & & I_{xx}S^2 & -I_{xz}S^2 & \\
& & & & & & & & & & & -I_{xx}S^2 & I_{zz}S^2 & \\
& & & & & & & & & & & & & \begin{array}{c}\overline{m}_A S^2\\ \overline{D}_A S\\ \overline{K}_A\end{array}
\end{array}\right]
\left[\begin{array}{c}
\bar{u}_p \\ \bar{w}_p \\ \bar{q}_p \\ \bar{x}' \\ \bar{z}' \\ \bar{\theta} \\ \bar{u}_{e_s} \\ \bar{v}_p \\ \bar{p}_p \\ \bar{r}_p \\ \bar{y}' \\ \bar{\phi} \\ \bar{\Psi} \\ \bar{u}_{e_A}
\end{array}\right]
=
\left[\begin{array}{c}
o \\ o \\ o \\ \bar{F}_x \\ \bar{F}_z \\ \bar{M}_y \\ \bar{Q}_s \\ o \\ o \\ o \\ \bar{F}_y \\ \bar{M}_x \\ \bar{M}_z \\ \bar{Q}_A
\end{array}\right]
$$

$$\alpha_p = \frac{w_p}{U_1} = S\frac{z'_p}{U_1} + \theta_p$$

$$\beta_p = \frac{v_p}{U_1} = S\frac{y'_p}{U_1} - \Psi_p$$

Figure 20. – Lagrangian Perturbation Equations for a Straight and Level Reference Condition

APPENDIX C

EQUATIONS OF MOTION TIME VARIANT COEFFICIENTS

The forces to gravity in the inertia axis systems are:

$$F_{x'} = -\, Mg \sin\theta_o$$

$$F_{y'} = Mg \sin\phi_o \cos\theta_o$$

$$F_{z'} = Mg \cos\phi_o \cos\theta_o$$

Relating the inertia axis to the body axis, yields the forces due to gravity in the body-fixed axis system

$$
\begin{aligned}
F_x &= \cos\theta \cos\psi \, F_{x'} + \cos\theta \sin\psi \, F_{y'} - \sin\theta \, F_{z'} \\
F_y &= \left(\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi\right) F_{x'} \\
&\quad + \left(\sin\phi \sin\theta \sin\psi + \cos\phi \cos\psi\right) F_{y'} \\
&\quad + \sin\phi \cos\theta \, F_{z'} \\
F_z &= \left(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi\right) F_{x'} \\
&\quad + \left(\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi\right) F_{y'} \\
&\quad + \cos\phi \cos\theta \, F_{z'}
\end{aligned}
$$

$$
\begin{aligned}
F_x &= \left(\cos\theta_1 - \sin\theta_1 \, \theta_p\right)\left(\cos\psi_1 - \sin\psi_1 \, \psi_p\right) Mg \sin\theta_o \\
&\quad + \left(\cos\theta_1 - \sin\theta_1 \, \theta_p\right)\left(\sin\psi_1 + \cos\psi_1 \, \psi_p\right) Mg \sin\phi_o \cos\theta_o \\
&\quad - \left(\sin\theta_1 + \cos\theta_1 \, \theta_p\right) Mg \cos\phi_o \cos\theta_o
\end{aligned}
$$

$$
\begin{aligned}
-F_{x_p} &= A_{u,\theta}\theta_p + A_{u,\psi}\psi_p \\
A_{u,\theta} &= -Mg \left[\sin\theta_1 \cos\psi_1 \sin\theta_o - \sin\theta_1 \sin\psi_1 \sin\phi_o \cos\theta_o \right. \\
&\qquad \left. - \cos\theta_1 \cos\phi_o \cos\theta_o\right] \\
A_{u,\psi} &= -Mg \left[\cos\theta_1 \sin\psi_1 \sin\theta_o + \cos\theta_1 \cos\psi_1 \sin\phi_o \cos\theta_o\right]
\end{aligned}
$$

$$Fy = -\Big[\left(\sin\phi_1 + \cos\phi_1\, \phi_p\right) \left(\sin\theta_1 + \cos\theta_1\, \theta_p\right) \left(\cos\psi_1 - \sin\psi_1\, \psi_p\right) - \left(\cos\phi_1 - \sin\phi_1\, \phi_p\right) \left(\sin\psi_1 + \cos\psi_1\, \psi_p\right) \Big] Mg \sin\theta_o$$

$$+ \Big[\left(\sin\phi_1 + \cos\phi_1\, \phi_p\right) \left(\sin\theta_1 + \cos\theta_1\, \theta_p\right) \left(\sin\psi_1 + \cos\psi_1\, \psi_p\right) + \left(\cos\phi_1 - \sin\phi_1\, \phi_p\right) \left(\cos\psi_1 - \sin\psi_1\, \psi_p\right) \Big] Mg \sin\phi_o \cos\theta_o$$

$$+ \Big[\left(\sin\phi_1 + \cos\phi_1\, \phi_p\right) \left(\cos\theta_1 - \sin\theta_1\, \theta_p\right) \Big] Mg \cos\phi_o \cos\theta_o$$

$$-F_{yp} = A_{v\theta}\, \theta_p + A_{v,\phi}\phi_p + A_{v,\psi}\, \psi_p$$

$$A_{v,\theta} = -Mg \Big[-\sin\phi_1 \cos\theta_1 \cos\psi_1 \sin\theta_o + \sin\phi_1 \cos\theta_1 \sin\psi_1 \sin\phi_o \cos\theta_o - \sin\phi_1 \sin\theta_1 \cos\phi_o \cos\theta_o \Big]$$

$$A_{v,\phi} = -Mg \Big[-\cos\phi_1 \sin\theta_1 \cos\psi_1 \sin\theta_o - \sin\phi_1 \sin\psi_1 \sin\theta_o + \cos\phi_1 \sin\theta_1 \sin\psi_1 \sin\phi_o \cos\theta_o - \sin\phi_1 \cos\psi_1 \sin\phi_o \cos\theta_o + \cos\phi_1 \cos\theta_1 \cos\phi_o \cos\theta_o \Big]$$

$$A_{v,\psi} = -Mg \Big[\sin\phi_1 \sin\theta_1 \sin\psi_1 \sin\theta_o + \cos\phi 1 \cos\psi_1 \sin\theta_o + \sin\phi_1 \sin\theta_1 \cos\psi_1 \sin\phi_o \cos\theta_o - \cos\phi_1 \sin\psi_1 \sin\phi_o \cos\theta_o \Big]$$

$$Fz = -\Big[\left(\cos\phi_1 - \sin\phi_1\, \phi_p\right) \left(\sin\theta_1 + \cos\theta_1\, \theta_p\right) \left(\cos\psi_1 - \sin\psi_1\, \psi_p\right) + \left(\sin\phi_1 + \cos\phi_1\, \phi_p\right) \left(\sin\psi_1 + \cos\psi_1\, \psi_p\right) \Big] Mg \sin\theta_o$$

$$+ \Big[\left(\cos\phi_1 - \sin\phi_1\, \phi_p\right) \left(\sin\theta_1 + \cos\theta_1\, \theta_p\right) \left(\sin\psi_1 + \cos\psi_1\, \psi_p\right) - \left(\sin\phi_1 + \cos\phi_1\, \phi_p\right) \left(\cos\psi_1 - \sin\psi_1\, \psi_p\right) \Big] Mg \sin\phi_o \cos\theta_o$$

$$+ \Big[\left(\cos\phi_1 - \sin\phi_1\, \phi_p\right) \left(\cos\theta_1 - \sin\theta_1\, \theta_p\right) \Big] Mg \cos\phi_o \cos\theta_o$$

$$-F_{z,p} = A_{w,\theta}\theta_p + A_{w,\phi}\phi_p + A_{w,\psi}\psi_p$$

$$A_{w,\theta} = -Mg\left[-\cos\phi_1\cos\theta_1\cos\psi_1\sin\theta_o + \cos\phi_1\cos\theta_1\sin\psi_1\sin\phi_o\cos\theta_o - \cos\phi_1\sin\theta_1\cos\phi_o\cos\theta_o\right]$$

$$A_{w,\phi} = -Mg\left[\sin\phi_1\sin\theta_1\cos\psi_1\sin\theta_o - \cos\phi_1\sin\psi_1\sin\theta_o - \sin\phi_1\sin\theta_1\sin\psi_1\sin\phi_o\cos\theta_o - \cos\phi_1\cos\psi_1\cos\phi_o\cos\theta_o - \sin\phi_1\cos\theta_1\cos\phi_o\cos\theta_o\right]$$

$$A_{w,\psi} = -Mg\left[\cos\phi_1\sin\theta_1\sin\psi_1\sin\theta_o - \sin\phi_1\cos\psi_1\sin\theta_o + \cos\phi_1\sin\theta_1\cos\psi_1\sin\phi_o\cos\theta_o + \sin\phi_1\sin\psi_1\sin\phi_o\cos\theta_o\right]$$

For the condition that

$$\theta_o = \phi_o = 0$$

then

$$A_{u,\theta} = Mg\cos\theta_1$$

$$A_{u,\psi} = 0$$

$$A_{v,\theta} = Mg\sin\phi_1\sin\theta_1$$

$$A_{v,\phi} = -Mg\cos\phi_1\cos\theta_1$$

$$A_{v,\psi} = 0$$

$$A_{w,\theta} = Mg\cos\phi_1\sin\theta_1$$

$$A_{w,\phi} = Mg\sin\phi_1\cos\theta_1$$

$$A_{w,\psi} = 0$$

From equation 6.5-17 (ref. 10)

$$
\begin{aligned}
\dot{x}' = {} & u_p\left(\cos\theta_1 - \sin\theta_1\,\theta_p\right)\left(\cos\psi_1 - \sin\psi_1\,\psi_p\right) \\
& + U_1\left[\left(\cos\theta_1 - \sin\theta_1\,\theta_p\right)\left(\cos\psi_1 - \sin\psi_1\,\psi_p\right) - \cos\theta_1\,\cos\psi_1\right] \\
& + v_\rho\left[\left(\sin\phi_1 + \cos\phi_1\,\phi_p\right)\left(\sin\theta_1 + \cos\theta_1\,\theta_p\right)\left(\cos\psi_1 - \sin\psi_1\,\psi_p\right) - \left(\cos\phi_1 - \sin\phi_1\,\phi_p\right)\right. \\
& \qquad \left.\left(\sin\psi_1 + \cos\psi_1\,\psi_p\right)\right] \\
& + V_1\left[\left(\sin\phi_1 + \cos\phi_1\,\phi_p\right)\left(\sin\theta_1 + \cos\theta_1\,\theta_p\right)\left(\cos\psi_1 - \sin\psi 1\,\psi_p\right) - \left(\cos\phi_1 - \sin\phi_1\,\phi_p\right)\right. \\
& \qquad \left.\left(\sin\psi_1 + \sin\psi_1\,\psi_p\right) - \sin\phi_1\,\sin\theta_1\,\cos\psi_1 + \cos\phi_1\,\sin\psi_1\right] \\
& + w_p\left[\left(\cos\phi_1 - \sin\phi_1\,\phi_p\right)\left(\sin\theta_1 + \cos\theta_1\,\theta_p\right)\left(\cos\psi_1 - \sin\psi_1\,\psi_p\right) + \left(\sin\phi_1 + \cos\phi_1\,\phi_p\right)\right. \\
& \qquad \left.\left(\sin\psi_1 + \cos\psi_1\,\psi_p\right)\right] \\
& + W_1\left[\left(\cos\phi_1 - \sin\phi_1\,\phi_p\right)\left(\sin\theta_1 + \cos\theta_1\,\theta_p\right)\left(\cos\psi_1 - \sin\psi_1\,\psi_p\right) + \left(\sin\phi_1 + \cos\phi_1\,\phi_p\right)\right. \\
& \qquad \left.\left(\sin\psi_1 + \cos\psi_1\,\psi_p\right) - \left(\cos\phi_1\,\sin\theta_1\,\cos\psi_1 - \sin\phi_1\,\sin\psi_1\right)\right]
\end{aligned}
$$

Then

$$
\begin{aligned}
\dot{x}' = {} & \left[A_{x'u}\,u_p + A_{x'w}\,w_p + A_{x'\theta}\,\theta_p + A_{x'v}\,v_p + A_{x'_\phi}\,\phi_p + A_{x'\psi}\,\psi_p\right. \\
= {} & u_p\left[\cos\theta_1\,\cos\psi_1\right] \\
& + w_p\left[\cos\phi_1\,\sin\theta_1\,\cos\psi_1 + \sin\phi_1\,\sin\psi_1\right] \\
& + \theta_p\left[-U_1\,\sin\theta_1\,\cos\psi_1 + V_1\,\sin\phi_1\,\cos\theta_1\,\cos\psi_1 + W_1\,\cos\phi_1\,\cos\theta_1\,\cos\psi_1\right] \\
& + v_p\left[\sin\phi_1\,\sin\theta_1\,\cos\psi_1 - \cos\phi_1\,\sin\psi_1\right] \\
& + \phi_p\left[V_1\,\cos\phi_1\,\sin\theta_1\,\cos\psi_1 + V_1\,\sin\phi_1\,\sin\psi_1 - W_1\,\sin\phi_1\,\sin\theta_1\,\cos\psi_1 + W_1\,\cos\phi_1\,\sin\psi_1\right] \\
& + \psi_p\left[-U_1\,\cos\theta_1\,\sin\psi_1 - V_1\,\sin\phi_1\,\sin\theta_1\,\sin\psi_1 - V_1\,\cos\phi_1\,\cos\psi_1 - W_1\,\cos\phi_1\,\sin\theta_1\,\sin\psi_1\right. \\
& \left.+ W_1\,\sin\phi_1\,\cos\psi_1\right]
\end{aligned}
$$

$$\dot{y}' = u_p\Big[\Big(\cos\theta_1 - \sin\theta_1\,\theta_1\Big)\Big(\sin\psi_1 + \cos\psi_1\,\psi_p\Big)\Big]$$

$$+ U_1\Big[\Big(\cos\theta_1 - \sin\theta_1\,\theta_p\Big)\Big(\sin\psi_1 + \cos\psi_1\,\psi_p\Big) - \cos\theta_1\sin\psi_1\Big]$$

$$+ v_p\Big[\Big(\sin\phi_1 + \cos\phi_1\,\phi_p\Big)\Big(\sin\theta_1 + \cos\theta_1\,\theta_p\Big)\Big(\sin\psi_1 + \cos\psi_1\,\psi_p\Big) + \Big(\cos\phi_1 - \sin\phi_1\,\phi_p\Big)$$

$$\Big(\cos\psi_1 - \sin\psi_1\,\psi_p\Big)\Big]$$

$$+ V_1\Big[\Big(\sin\phi_1 + \cos\phi_1\,\phi_p\Big)\Big(\sin\theta_1 + \cos\theta_1\,\theta_p\Big)\Big(\sin\psi_1 + \cos\psi_1\,\psi_p\Big) + \Big(\cos\phi_1 - \sin\phi_1\,\phi_p\Big)$$

$$\Big(\cos\psi_1 - \sin\psi_1\,\psi_p\Big) - \sin\phi_1\sin\theta_1\sin\psi_1 - \cos\phi_1\cos\psi_1\Big]$$

$$+ w_p\Big[\Big(\cos\phi_1 - \sin\phi_1\,\phi_p\Big)\Big(\sin\theta_1 + \cos\theta_1\,\theta_p\Big)\Big(\sin\psi_1 + \cos\psi_1\,\psi_p\Big) - \Big(\sin\phi_1 + \cos\phi_1\,\phi_p\Big)$$

$$\Big(\cos\psi_1 - \sin\psi_1\,\psi_p\Big)\Big]$$

$$+ W_1\Big[\Big(\cos\phi_1 - \sin\phi_1\,\phi_p\Big)\Big(\sin\theta_1 + \cos\theta_1\,\theta_p\Big)\Big(\sin\psi_1 + \cos\psi_1\,\psi_p\Big) - \Big(\sin\phi_1 + \cos\phi_1\,\phi_p\Big)$$

$$\Big(\cos\psi_1 - \sin\psi_1\,\psi_p\Big) - \cos\phi_1\sin\theta_1\sin\psi_1 + \sin\phi_1\cos\psi_1\Big]$$

$$\dot{y}' = A_{y'u}\,u_p + A_{y'w}\,w_p + A_{y'\theta}\,\theta_p + A_{y'v}\,v_p + A_{y'\phi}\,\phi_p + A_{y'\psi}\,\psi_p$$

$$\dot{y}' = u_p\Big[\cos\theta_1\sin\psi_1\Big]$$

$$+ v_p\Big[\sin\phi_1\sin\theta_1\sin\psi_1 + \cos\phi_1\cos\psi 1\Big]$$

$$+ \theta_p\Big[-U_1\sin\theta_1\sin\psi_1 + V_1\sin\phi_1\cos\theta_1\sin\psi_1 + W_1\cos\phi_1\cos\theta_1\sin\psi_1\Big]$$

$$+ w_p\Big[\cos\phi_1\sin\theta_1\sin\psi_1 - \sin\phi_1\cos\psi_1\Big]$$

$$+ \phi_p\Big[V_1\cos\phi_1\sin\theta_1\sin\psi_1 - V_1\sin\phi_1\cos\psi_1 - W_1\sin\phi_1\sin\theta_1\sin\psi_1 - W_1\cos\phi_1\cos\psi_1\Big]$$

$$+ \psi_p\Big[U_1\cos\theta_1\cos\psi_1 + V_1\sin\phi_1\sin\theta_1\cos\psi_1 - V_1\cos\phi_1\sin\psi_1 + W_1\cos\phi_1\sin\theta_1\cos\psi_1$$

$$+ W_1\sin\phi_1\sin\psi_1\Big]$$

$$\mathring{z}' = -u_p\left[\sin\theta_1 + \cos\theta_1\,\theta_p\right]$$

$$-U_1\left[\sin\theta_1 + \cos\theta_1\,\theta_p - \sin\theta_1\right]$$

$$+ v_p\left[(\sin\phi_1 + \cos\phi_1\,\phi_p)(\cos\theta_1 - \sin\theta_1\,\theta_p)\right]$$

$$+ V_1\left[(\sin\phi_1 + \cos\phi_1\,\phi_p)(\cos\theta_1 - \sin\theta_1\,\theta_p) - \sin\phi_1\cos\theta_1\right]$$

$$+ w_p\left[(\cos\phi_1 - \sin\phi_1\,\phi_p)(\cos\theta_1 - \sin\theta_1\,\theta_p)\right]$$

$$+ W_1\left[(\cos\phi_1 - \sin\phi_1\,\phi_p)(\cos\theta_1 - \sin\theta_1\,\theta_p) - \cos\phi_1\cos\theta_1\right]$$

$$z' = u_p\left[-\sin\theta_1\right]$$

$$+ w_p\left[\cos\phi_1\cos\theta_1\right]$$

$$+ \theta_p\left[-u_1\cos\theta_1 - V_1\sin\phi_1\sin\theta_1 - W_1\cos\phi_1\sin\theta_1\right]$$

$$+ v_p\left[\sin\phi_1\cos\theta_1\right]$$

$$+ \phi_p\left[V_1\cos\phi_1\cos\theta_1 - W_1\sin\phi_1\cos\theta_1\right]$$

$$+ \psi_p\,[0]$$

$$= A_{z'u}\,u_p + A_{z'w}\,w_p + A_{z'\theta}\,\theta_p + A_{z'v}\,v_p + A_{z'\phi}\,\phi_p$$

However, for rectilinear flight, the following must be true

$$\mathring{\phi}_1 = P_1 + \left(Q_1\sin\phi_1 + R_1\cos\phi_1\right)\tan\theta_1 = 0$$

$$\mathring{\theta}_1 = Q_1\cos\phi_1 - R_1\sin\phi_1 = 0$$

$$\mathring{\psi}_1 = \left(Q_1\sin\phi_1 + R_1\cos\phi_1\right)\sec\theta_1 = 0$$

$$\therefore P_1 = Q_1 = R_1 = 0$$

APPENDIX D

DERIVATION OF PERTURBATION AERODYNAMICS FORCES FOR THE INERTIA AXIS SYSTEM

The following shows the symmetric perturbation forces expanded in terms of the derivatives of the forces X', Z' and M_y' with respect to the inertia axis coordinates (Q). Note that δ is a control surface rotation. The force derivatives are found in terms of the aerodynamic derivatives using the expressions in figure 15 and neglecting products of small perturbations. It should be noted that the aerodynamic derivatives are those used in reference 10, and equations (A-1) are used to transform the coordinates from body to inertia. Also, the reference drag coefficient C_{D_1} is understood to contain thrust and consequently is zero.

The perturbation forces are expanded in terms of the inertia axis coordinates (Q) thus

$$F_x' = \sum \frac{\partial X'}{\partial Q} Q = \frac{\partial X'}{\partial \dot{x}'} \dot{x}' + \frac{\partial X'}{\partial \dot{z}'} \dot{z}' + \frac{\partial X'}{\partial \ddot{z}'} \ddot{z}' + \frac{\partial X'}{\partial \theta'} \theta' + \frac{\partial X'}{\partial \dot{\theta}'} \dot{\theta}'$$

$$+ \frac{\partial X'}{\partial \ddot{\theta}'} \ddot{\theta}' + \frac{\partial X'}{\partial \delta} \delta + \frac{\partial X'}{\partial \dot{\delta}} \dot{\delta}$$

$$F_z' = \sum \frac{\partial Z'}{\partial Q} Q = \frac{\partial Z'}{\partial \dot{x}'} \dot{x}' + \text{Etc.}$$

$$M_y' = \sum \frac{\partial M_y'}{\partial Q} Q = \frac{\partial M_y'}{\partial \dot{x}'} \dot{x}' + \text{Etc.}$$

where (from the equations in figure A3):

$$\frac{\partial X'}{\partial Q} = \frac{\partial L}{\partial Q}\left(\alpha_2 + \frac{\dot{z}'}{U_1}\right) + L \frac{\partial\left(\alpha_1 + \frac{\dot{z}'}{U_1}\right)}{\partial Q} - \frac{\partial D}{\partial Q}$$

$$\frac{\partial Z'}{\partial Q} = -\frac{\partial L}{\partial Q} - \frac{\partial D}{\partial Q}\left(\alpha_1 + \frac{\dot{z}'}{U_1}\right) - D \frac{\partial\left(\alpha_1 + \frac{\dot{z}'}{U_1}\right)}{\partial Q}$$

$$\frac{\partial M_y'}{\partial Q} = \frac{\partial m}{\partial Q}$$

The lift, drag, and moment are expanded in terms of the body-fixed axis coordinates thus

$$L = \bar{q} S_W \left(C_{L_1} + C_{L_{\hat{u}}} \frac{u}{U_1} + C_{L_\alpha} \frac{w}{U_1} + C_{L_{\hat{\dot{\alpha}}}} \frac{\bar{c}}{2U_1} \frac{\dot{w}}{U_1} \right.$$

$$\left. + C_{L_q} \frac{\bar{c}}{2U_1} q + C_{L_{\hat{\dot{q}}}} \frac{\bar{c}^2}{2U_1^2} \dot{q} + C_{L_\delta} \delta + C_{L_{\hat{\dot{\delta}}}} \frac{\bar{c}}{2U_1} \dot{\delta} \right)$$

$$D = \bar{q} S_W \left(\cancelto{0}{C_{D_1}} + C_{D_{\hat{u}}} \frac{u}{U_1} + \text{Etc.} \right.$$

$$m = \bar{q} S_W \bar{c} \left(C_{m_1} + c_{m_{\hat{u}}} \frac{u}{U_1} + \text{Etc.} \right.$$

or – substituting from equations (A1):

$$u = \dot{x}' - W_1 \theta'$$
$$w = \dot{z}' + U_1 \theta'$$
$$q = \theta'$$

The expansion in terms of the inertia axis coordinates is:

$$L = \bar{q} S_W \left[C_{L_1} + C_{L_{\hat{u}}} \frac{\dot{x}'}{U_1} + C_{L_\alpha} \frac{\dot{z}'}{U_1} + C_{L_{\hat{\dot{\alpha}}}} \frac{\bar{c}}{2U_1} \frac{\ddot{z}'}{U_1} + \left(-C_{L_{\hat{u}}} \alpha^*_1 + C_{L_\alpha} \right) \theta' \right.$$

$$+ \left(C_{L_{\hat{\dot{\alpha}}}} \frac{\bar{c}}{2U_1} + C_{L_{\hat{q}}} \frac{\bar{c}}{2U_1} \right) \dot{\theta}'$$

$$\left. + C_{L_{\hat{\dot{q}}}} \frac{\bar{c}^2}{2U^2} \ddot{\theta}' + C_{L_\delta} \delta + C_{L_{\hat{\dot{\delta}}}} \frac{\bar{c}}{2U_1} \dot{\delta} \right]$$

$$D = \bar{q} S_W \left[\cancelto{0}{C_{D_1}} + C_{D_{\hat{u}}} \frac{\dot{x}'}{U_1} + \text{Etc.} \right.$$

$$m = \bar{q} S_W \bar{c} \left[C_{m_1} + C_{m_{\hat{u}}} \frac{\dot{x}'}{U_1} + \text{Etc.} \right.$$

$$^*\alpha_1 = \frac{W_1}{U_1}$$

Finally

$$F_{x}' = \left[\frac{\partial L}{\partial \dot{x}'}\left(\alpha_1 + \frac{\dot{z}'}{U_1}\right) + L \times 0 - \frac{\partial D}{\partial \dot{x}'}\right]\dot{x}' + \left[\frac{\partial L}{\partial \dot{z}'}\left(\alpha_1 + \frac{\dot{z}'}{U_1}\right) + L \times \frac{1}{U_1} - \frac{\partial D}{\partial \dot{z}'}\right]\dot{z}'$$

$$+ \left[\frac{\partial L}{\partial \ddot{z}'}\left(\alpha + \frac{\dot{z}'}{U_1}\right) + L \times 0 - \frac{\partial D}{\partial \ddot{z}'}\right]\ddot{z}' + \left[\frac{\partial L}{\partial \theta'}\left(\alpha_1 + \frac{\dot{z}'}{U_1}\right) + L \times 0 - \frac{\partial D}{\partial \theta'}\right]\theta' \quad + \text{similar terms in}$$

$\dot{\theta}', \ddot{\theta}'$, δ, and $\dot{\delta}$

Neglecting products of small perturbation such as $\dot{z}'\,\dot{x}'$, $\dot{z}'\,\dot{z}'$, etc. and those which arise in the second term in the expansion of $L \times \frac{1}{U_1}\dot{z}'$:

$$F_x' = \left[\frac{\bar{q}\,S_W}{U_1}\,C_{L_{\hat{u}}}\,\alpha_1 - \frac{\bar{q}\,S_W}{U_1}\,C_{D_{\hat{u}}}\right]\dot{x}'$$

$$+ \left[\frac{\bar{q}\,S_W}{U_1}\,C_{L_\alpha}\alpha_1 + \frac{\bar{q}\,S_W}{U_1}\,C_{L_1} - \frac{\bar{q}\,S_W}{U_1}\,C_{D_\alpha}\right]\dot{z}'$$

$$\left[\frac{\bar{q}\,S_W\,\bar{c}}{2U_1^2}\,C_{L_{\hat{\dot{\alpha}}}}\,\alpha_1 - \frac{\bar{q}\,S_W\,\bar{c}}{2U_1^2}\,C_{D_{\hat{\dot{\alpha}}}}\right]\ddot{z}'$$

$$+ \left[\bar{q}\,S_W\left(C_{L_\alpha} - \alpha_1\,C_{L_{\hat{u}}}\right)\alpha_1 - \bar{q}\,S_W\left(C_{D_\alpha} - \alpha_1\,C_{D_u}\right)\right]\theta'$$

$$+ \left[\frac{\bar{q}\,S_W\,\bar{c}}{2U_1}\left(C_{L_{\hat{\dot{\alpha}}}} + C_{L_{\hat{q}}}\right)\alpha_1 - \bar{q}\,S_W\,\frac{\bar{c}}{2U_1}\left(C_{D_{\hat{\dot{\alpha}}}} + C_{D_{\hat{q}}}\right)\right]\dot{\theta}'$$

$$+ \left[\frac{\bar{q}\,S_W\,\bar{c}^2}{2U_1}\,C_{L_{\hat{\dot{q}}}}\,\alpha_1 - \frac{\bar{q}\,S_W\,\bar{c}^2}{2U_1}\,C_{D_{\hat{\dot{q}}}}\right]\ddot{\theta}'$$

$$+ \left[\bar{q}\,S_W\,C_{L_\delta}\,\alpha_1 - \bar{q}\,S_W\,C_{D_\delta}\right]\delta$$

$$+ \left[\bar{q}\,S_W\,\frac{\bar{c}}{2U_1}\,C_{L_{\hat{\dot{\delta}}}}\,\alpha_1 - \bar{q}\,S_W\,\frac{\bar{c}}{2U_1}\,C_{D_{\hat{\dot{\delta}}}}\right]\dot{\delta}'$$

and similarly for F_z' and M_y'

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16. Abstract

L219 (EQMOD) is a digital computer program available for execution on the CDC 6600. The program modifies matrices according to card input instructions and prepares files of matrices suitable for use in the Linear Systems Analysis program (QR) and the Random Harmonic Analysis program, L221 (TEV156).

The particular field of application of the program is the modification of the theoretical equations of motion and load equations generated in DYLOFLEX by the Equations of Motion program (L217) and the Load Equation program (L218), respectively.

Program usage and a brief description of the analysis used are presented in volume I of this document. Volume II contains a description of the design and structure of the program to aid those persons who will maintain and/or modify the program in the future.

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